

# Plundering Coalitions\*

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## Abstract

We develop a model to study coalitions that extract the resources of outsiders. The players in our model are endowed with power and resources. The ruling coalition plunders outsiders, distributes the plundered resources among its members, and guarantees that insiders' resources remain safe. Our analysis focuses on the resilience of the equilibrium ruling coalition to exogenous shocks affecting the power and resources of both insiders and outsiders, as well as the intensity of plundering. We show that a coalition with a classical hierarchical structure—where power and resources are equal within each “rank” but strictly higher in higher ranks—produces greater resilience to external shocks affecting outsiders' power and resources. The only exception arises when plundering intensity is relatively weak, in which case the internal distribution of power and resources does not affect external resilience. Our final results provide insights into how the intensity of plundering impacts the internal and external resilience of ruling coalitions across political environments.

**Keywords:** Political Economy, Coalition Formation, Institutions, Resilience, Plundering.

## 1 Introduction

Coalition formation is always challenging (Ray and Vohra (2015a)), and a “plundering coalition” is no exception. For such a coalition, the wealth they can distribute among

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coalition members is plundered from the outsiders. This setup applies to a wide range of important social phenomena, such as an army that plunders the civil society, or an oligarchical government that extracts from its citizens (Puga and Trefler (2014); Xu (2018); Sánchez De La Sierra (2020); Henn et al. (2024)). We formally study the problem to form a coalition whose primary objective is to plunder outsiders. To our knowledge, this is the first such attempt in coalition formation games. Our model yields a series of novel results. Among others, we study the resilience of a plundering coalition against outsiders, which justifies the key organizational principle of hierarchy for an effective army or a stable oligarchy. We also propose a new methodology to analyze the resilience of an equilibrium coalition against exogenous shocks.

Our model features a society of finitely many individuals, each endowed with power and wealth. A coalition is “winning” if its aggregate *power* exceeds a supermajority threshold.<sup>1</sup> The game opens with an initial winning coalition, whose members sequentially propose alternative coalitions. A proposal is adopted—and the proposed coalition becomes the ruling coalition—if it commands a supermajority of power and is accepted by all of its members. If no proposal succeeds, the initial coalition rules by default. The ruling coalition then defeats the outsiders, plunders their wealth, and distributes the spoils among its members.

We are primarily interested in the properties of the ruling coalition. The ruling coalition is shaped by the following trade-off: admitting a new member raises the power of the coalition, which can plunder more wealth from defeated outsiders; but the new member is also costly because the coalition cannot plunder the wealth of the new member anymore. We show that the ruling coalition which optimally balances this trade-off exists, is unique, and admits a clean axiomatic characterization.

For the equilibrium ruling coalition, we then characterize a necessary and sufficient condition, which prepares our central analysis of coalitional resilience. The ruling coalition must outperform, in terms of plunder, two classes of alternatives: its own sub-coalitions, and any alliance between one of its sub-coalitions and a subset of outsiders. These two conditions motivate two distinct notions of resilience. A coalition is more internally resilient if it is more likely to survive exogenous shocks to the power and resources of its own members. It is more externally resilient if, holding its members’ characteristics fixed, it is more likely to survive exogenous shocks to the power and resources of outsiders.

To understand the conditions of high external resilience, we conduct a thought experiment. Take any two coalition members and make them more homogeneous: transfer power from the stronger to the weaker, or wealth from the richer to the poorer, without

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<sup>1</sup>Formally, a coalition is winning if its aggregate power exceeds  $\beta$  fraction of society’s total power, with  $\beta > 1/2$ . Notice that here the super-majority refers to power, not to votes.

flipping their relative ranks. This transfer holds the characteristics of the ruling coalition constant, so it is still the unique ruling coalition. But importantly, such a transfer reduces the risk of the more threatening member with stronger power or lower wealth. After the transfer, the ruling coalition becomes more resilient to any alliance between a sub-coalition that includes the more threatening member and any subset of outsiders, where the outsiders are subject to any possible perturbation of their resources and power. At the same time, the ruling coalition is equally resilient to an alliance between a sub-coalition that includes the less threatening member and any subset of outsiders. Therefore, the ruling coalition becomes more externally resilient if two of its members become more homogeneous.

It is important to note that the analysis does not imply that absolute equality maximizes external resilience. Instead, the analysis implies that more externally resilient than others is a ruling coalition of a classic hierarchical structure. Such a hierarchical coalition is partitioned into well-defined “ranks.” Within each “rank,” all members are absolutely equal with each other; but higher “ranked” members are both richer and more powerful than lower ranked members. Once such a hierarchy emerges, it is not possible to further improve external resilience through an operation of transfer as above. Our analysis therefore offers a justification for the classical hierarchical structure of many organizations, such as armies and bureaucracy, by their unique capacity in bearing changes to its enemies/subjects. This justification is, as far as we know, novel, in contrast to the conventional emphasis on the advantage of a hierarchical structure in incentive-alignment (Qian (1994); Mookherjee (2013)) or division of labor in (Garicano (2000); Garicano and Rossi-Hansberg (2015)). Finally, we jointly investigate how internal and external resilience respond to a shift in the plundering technology. Specifically, suppose that the plundering technology can now extract more wealth from outsiders, holding all else constant. A more powerful plundering technology raises the cost of keeping any player inside the coalition, since insiders’ wealth are shielded from extraction. This tilts members’ preferences toward exclusive alternatives—sub-coalitions that are smaller, less powerful, and poorer—and away from inclusive ones. Since the internal threats to the ruling coalition are precisely these exclusive sub-coalitions, a more effective plundering technology ironically erodes the coalition’s internal resilience. Plundering more intensively, in short, makes the coalition harder to hold together.

For the external resilience of the ruling coalition, a stronger plundering technology is a double-edged sword. On one hand, exclusive alternatives which involve small segments of society become more threatening to the ruling coalition. On the other hand, inclusive alternatives which encompass broader segments of society become less threatening. Thus, the realization of these alternatives—the specification of shocks—becomes particularly

important. If the exclusive alternatives are more likely to emerge, a stronger plundering technology decreases external resilience. Instead, if inclusive alternatives are more likely to appear, a stronger plundering process increases the external resilience. The latter suggests that a ruling coalition that engages in power-light plundering of society benefit more from facing a more powerful and wealthier opposition than a weaker and poorer one.

Lastly, although the direction of change in external resilience generally depends on the realization of powers and resources inside the ruling coalition, we identify a wide range of political environments where this is not the case. That is, the external resilience is robust with respect to changes in internal configuration of powers and resources. In these political environments, the plundering process is “power-intensive;” for instance, it is endowed with better protections of property rights. Precisely, in these contexts, corresponding to any exclusive alternative, there always exists an inclusive alternative that is more threatening to the ruling coalition. This implies that the only factor affecting external resilience is the players’ preference for inclusive alternatives. As a result, a stronger plundering technology always increases the external resilience of the ruling coalition, since it renders the inclusive alternatives less beneficial for the players. Thus, in power-intensive plundering environments, there exists a trade-off between external and internal resilience of the ruling coalition with respect to the plundering intensity, regardless of the specifications of internal and external shocks. This offers a novel insight: even imperfect property rights—which do not fully prevent plundering by insiders—could potentially hinder the ruling coalition from achieving both internal and external stability when plundering technology changes. This contrasts with “power-light plundering” environments, wherein a change in plundering technology could alleviate both internal and external threats to the ruling coalition.

## 1.1 Relevant Literature

Our paper is relevant to a few strands of literature. The literature on coalition formation largely focus on characterizing the equilibrium coalition ([Acemoglu et al. \(2008\)](#); [Ray and Vohra \(2015b\)](#); [Battaglini \(2021\)](#)), or define stability mainly by incorporating the notion of “farsightedness” ([Harsanyi \(1974\)](#); [Ray and Vohra \(2015c\)](#)). We instead take one step further by analyzing the resilience of the equilibrium coalition against exogenous shocks. By doing so, we make a methodological contribution by proposing a simple framework to analyze the resilience of the equilibrium coalition. This novel focus on resilience also uncovers a lot of new substantive insights.

We bring together the two strands of literature on coalition formation and organizational economics of hierarchy. Existing literature usually focus on how a hierarchy

may improve incentive-alignment or division of labor (Qian (1994); Qian et al. (2006); Mookherjee (2013); Garicano (2000); Garicano and Rossi-Hansberg (2015)). We offer a new justification for hierarchy: we show that a hierarchy is uniquely resistant to arbitrary exogenous changes to the characteristics of individuals outside the hierarchy. Our novel justification might be relevant to many hierarchies where the characteristics of outsiders are a first order concern, such as armies and fiscal bureaucracies (Besley and Persson (2009); Xu (2018); Sánchez De La Sierra (2020); Henn et al. (2024)).

Our model also makes novel contributions to a few central debates in political economy. First, political economists have uncovered that the interaction between power and wealth is a fundamental thread in political economy (Acemoglu and Robinson (2008); Dal Bó and Dal Bó (2011); Dal Bó et al. (2022); Acemoglu and Robinson (2013)). We contribute to this literature by an in-depth analysis of the power-wealth trade-off through the lens of coalition formation, the first ever attempt to our knowledge. It is through the coalition analysis that we uncover the innovative insight on the unique resilience of a hierarchical organization.

Our analysis also contributes to the burgeoning literature on political economy of non-democracies (Egorov and Sonin (2024)). Specifically, our analysis of internal and external resilience engages with the literature that addresses the trade-offs that authoritarian states resolve while dealing with internal or external threats to their rule. A strand of literature studies the loyalty-competence trade-off, i.e., how autocratic states balance the competence of their officials against their loyalty to prevent internal dissent (Besley and Kudamatsu (2007); Egorov and Sonin (2011); Jia et al. (2015); Zakharov (2016); Bai and Zhou (2019); Mattingly (2024)). Another strand of literature focuses on external problems such as mass protests, or propaganda (Wintrobe (1990); Wintrobe (2000); Konrad and Skaperdas (2007); Egorov et al. (2009); De Mesquita (2010); Yanagizawa-Drott (2014); Shadmehr (2018)). There are many trade-offs the dictators resolve while tackling external threats, for instance, the one between “informational openness” and “security” (Lorentzen (2013); Gehlbach and Sonin (2014); Lorentzen (2014); Guriev and Treisman (2019); Enikolopov et al. (2020)). Through the novel lens of coalition formation, we contribute to this literature by showing how the internal and external threats are related. In particular, we identify the condition for a trade-off between internal and external resilience driven by the process of coalition formation. Additionally, we provide insights into when this trade-off does not hold, and the characteristics of oppositions that can benefit an autocratic state engaging in intensive plundering.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the preliminary analysis of the coalition formation game. Building on Section 3, we proceed by studying the resilience in Section 4. Section 5 concludes.

## 2 Environment

The society consists of a finite set of players  $N = \{1, 2, \dots, n\}$ , and  $2^N$  denote the set of all subsets of  $N$ . Time is finite and indexed by  $t \in \{1, 2, \dots, T\}$ . The players are endowed with a pair of power  $p$  and resources  $x$ , specified by the mappings

$$\begin{aligned} p(\cdot) &: N \rightarrow \mathbb{R}_{++}, \\ x(\cdot) &: N \rightarrow \mathbb{R}_{++}. \end{aligned}$$

We refer to  $p_i := p(i)$  and  $x_i := x(i)$  as the political power and resources of individual  $i \in N$ . A coalition is any non-empty subset  $I \subseteq N$ , and each player belongs to at most one coalition at any stage of the game. The power and resources of any coalition  $I \subseteq N$  are

$$P_I := \sum_{i \in I} p_i \quad \text{and} \quad X_I := \sum_{i \in I} x_i.$$

In particular,  $P_N := \sum_{i \in N} p_i$  and  $X_N := \sum_{i \in N} x_i$  are the power and resources of the society. A coalition  $I$  is a winning coalition if  $P_I \geq \beta P_N$ , where  $\beta \in (1/2, 1]$  is a fixed supermajority requirement for power so that the winning coalition can defeat all outsiders. Note again that the supermajority requirement applies to power, not to votes. Denote the set of all winning coalitions by  $\mathcal{W}$ . There is a baseline payoff function  $U : N \times \mathcal{W} \rightarrow \mathbb{R}$  that, for any player  $i \in N$ , assigns the payoff  $U_i(I)$  if the winning coalition  $I \in \mathcal{W}$  becomes the “ruling coalition.” We also write  $U(i, I) := U_i(I)$ .

A ruling coalition of our model is necessarily a winning coalition. As the key novelty of our setup, a ruling coalition can only plunder outsiders, while the resources of its members are safe. This creates a central trade-off for our model. A new member who is brought into the ruling coalition strengthens its capability to plunder outsiders, but the ruling coalition loses the opportunity to plunder this new member. This key trade-off is formally captured by Assumption 1.

**Assumption 1.** *[Payoffs] For any  $i \in N$  and  $I \in \mathcal{W}$ ,  $U_i(I) := x_i + w_i(I)$ , where  $w_i(\cdot)$  satisfies the following properties:*

1. (Trade-off) *If  $I \in \mathcal{W} \setminus \{N\}$  and  $i \in I$ , we have  $w_i(I) = G_i(P_I, X_I) > 0$ , where  $G_i(\cdot, \cdot) : [\beta P_N, P_N) \times [0, X_N) \rightarrow \mathbb{R}_{++}$  is a function continuous in  $P_I$  and  $X_I$ , satisfying the following conditions.*
  - (a) *For all  $I$  and  $I' \in \mathcal{W} \setminus \{N\}$  with  $P_I = P_{I'}$ , if  $i \in I$  and  $i \in I'$ , then  $G_i(P_I, X_I) > G_i(P_{I'}, X_{I'})$  if and only if  $X_I < X_{I'}$ .*
  - (b) *For all  $I$  and  $I' \in \mathcal{W} \setminus \{N\}$  with  $X_I = X_{I'}$ , if  $i \in I$  and  $i \in I'$ , then  $G_i(P_I, X_I) > G_i(P_{I'}, X_{I'})$  if and only if  $P_I > P_{I'}$ .*

2. If  $I \in \mathcal{W} \setminus \{N\}$  and  $i \notin I$ , then  $w_i(I) < 0$ .

3. For all  $i \in N$ ,  $w_i(N) = 0$ .

Assumption 1 establishes important primitives of the model. Part 1 introduces the key primitive, the function

$$G_i(\cdot, \cdot).$$

The function  $G_i(\cdot, \cdot)$  ranks the plundered resources of any individual across non-trivial ruling coalitions of which she is a member.<sup>2</sup> Part 1(a) says that between ruling coalitions with equal power, players prefer the one with fewer internal resources, which permits more external resources for plundering. Meanwhile, between ruling coalitions with equal resources, players prefer the one with larger power (Part 1(b)), as it strengthens the ruling coalition in extracting resources. Both Part 1(a) and Part 1(b) imply that when the ruling coalition is not the grand coalition  $N$ , insiders obtain strictly positive payoffs from plundering outsiders. Together with Part 2, this implies that inclusion in the ruling coalition strictly benefits insiders relative to their initial resources, while exclusion strictly harms outsiders relative to their initial resources. Part 3 states that the players' payoff from the plundered resources is zero when the ruling coalition is  $N$ , since there are no outsiders to plunder.

Under Assumption 1, a ruling coalition is fully characterized by its power and resources. This keeps the model tractable by eliminating the complexities that arise when the specific combination of players inside the ruling coalition also matters. For the rest of this paper, we thus write  $G_i(P_I, X_I)$  for player  $i$ 's plunder gains in coalition  $I$ . Assumption 1 immediately yields the following lemma.

**Lemma 1.** *Under Assumption 1, any player  $i \in N$  has strictly increasing and continuous indifference curves over  $(P, X)$ . The variables  $P$  and  $X$  are the aggregate power and resources of ruling coalitions that include the player  $i$ ;  $(P, X) \in [\beta P_N, P_N] \times [0, X_N]$ .*

The following assumption imposes essentially common preferences for players over coalitions, which simplifies notations throughout the paper. We later show that the main results continue to hold under a considerably weaker assumption.

**Assumption 2.** *For all  $I \in \mathcal{W}$  and all  $i \in I$ ,  $G_i(P_I, X_I) := g(i)G(P_I, X_I)$ , where  $g(i) > 0$ .*

Under Assumption 2, there are two components in a player's preference over ruling coalitions that include the player: an idiosyncratic component  $g(i)$  and a common

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<sup>2</sup>For example, one can view  $G_i(P_I, X_I)$  as a combination of a plundering component  $F(I) : \mathcal{W} \rightarrow \mathbb{R}_{++}$  and a share component  $\Pi(i, I) : N \times \mathcal{W} \rightarrow [0, 1]$ , i.e.,  $G_i(P_I, X_I) := \Pi(i, I)F(I)$  is the share allocated to individual  $i$  within the coalition  $I$  from plundered resources  $F(I)$ .

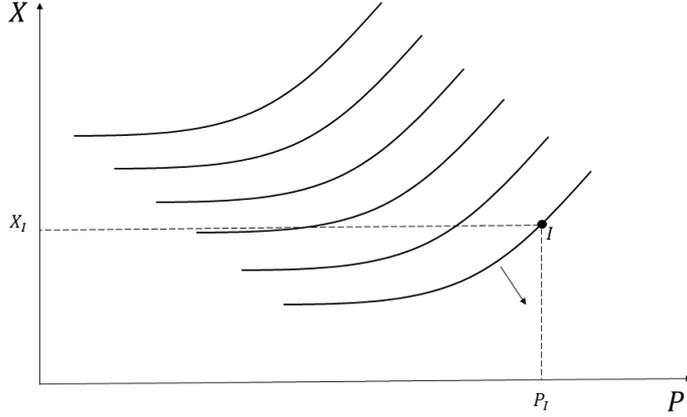


Figure 1: Identical indifference curves under Assumption 2

component  $G(P_I, X_I)$ , which depends on the aggregate powers and resources of the coalition,  $(P_I, X_I)$ . This assumption implies that for all players, the indifference curves over the coalitions containing them are the same and determined by the function  $G(\cdot)$  (Figure 1). In other words, for any ruling coalitions  $I, I' \in \mathcal{W}$  and any  $i, j \in I \cap I'$ , we have  $U_i(I) \geq U_i(I')$  if and only if  $U_j(I) \geq U_j(I')$ , i.e., the preferences of players over any pair of ruling coalitions including them are identical. We slightly abuse the notation and denote:

$$G(I) := G(P_I, X_I),$$

The function  $G(I)$  is particularly useful to characterize equilibrium coalition and its resilience.

**Discussion on Assumption 2.** Appendix B microfounds Assumption 2. It holds, for instance, when insiders' payoffs decompose as  $w_i(I) = \Pi_i(I)F(I)$ , where  $\Pi_i(I)$  is an intra-coalition share (e.g., proportional to  $p_i/P_I$ ) and  $F(I)$  is the coalition's total extractable surplus, depending only on aggregate characteristics  $(P_I, X_N - X_I)$ .

Assumption 2 is also not essential. All results carry through under a weaker condition requiring preference consistency only on the set of “potential ruling coalitions”  $Z$ , defined later in Definition 3.1.<sup>3</sup> This weaker condition is also natural: since we focus on the survival of the ruling coalition under external shocks, it is reasonable to require that members weakly prefer to be in that coalition before any shock occurs—otherwise external resilience is trivially zero.

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<sup>3</sup>As we will see, the proofs invoke preference comparisons only among coalitions in  $Z$  and the sub-coalitions and deviation coalitions generated by  $Z$  that enter the resilience constraints.

**Definition 1.** Fix a function  $G(\cdot, \cdot)$  that satisfies Assumption 1. For any ruling coalition  $I$ , denote the indifference curve through  $I$  by  $X := H_I(P)$ , which is implicitly defined by  $G(P, X) = G(P_I, X_I)$ .

Throughout the paper, we assume that the joint power and resources mapping is generic in the sense that for all  $I, I' \in \mathcal{W}$ , we have  $P_I \neq P_{I'}$  or  $X_I \neq X_{I'}$ .<sup>4</sup> The following assumption helps establish the uniqueness results in the subsequent section.

**Assumption 3.** Fix the power and resource mappings. Then, for all  $I, I' \in \mathcal{W}$ , we have  $G(I) \neq G(I')$ .<sup>5</sup>

This assumption implies that players receive strictly different payoffs from different ruling coalitions involving them.

### 3 Preliminary Analysis of the Coalition Formation Game

This section establishes existence and uniqueness of the coalitional equilibrium, which prepares our analysis of its “resilience,” i.e., how the equilibrium responds to exogenous shocks to players’ power or resources.

#### 3.1 Axiomatic analysis

We begin with an axiomatic analysis. As in Acemoglu et al. (2008) and Acemoglu et al. (2012), our axiomatic analysis shows that our results are independent of the details of the agenda-setting and voting protocols in the non-cooperative game introduced in Section 3.2. The axiomatic analysis will also help characterize the equilibrium of the non-cooperative game in Section 3.2.

Define a correspondence  $\phi : \mathcal{W} \rightrightarrows 2^N$ , which identifies the set of ruling coalitions corresponding to each initial winning coalition. We assume that  $\phi$  satisfies the following axioms:

**Axiom 1** (Non-triviality). For any  $I \in \mathcal{W}$ ,  $\emptyset \notin \phi(I)$  and  $N \notin \phi(I)$ .

**Axiom 2** (Super-majority of Power). For any  $I \in \mathcal{W}$  and any  $I' \in \phi(I)$ , we have  $I' \in \mathcal{W}$ .

<sup>4</sup>Mathematically, this assumption is without much loss of generality, since the set of vectors  $\{(P_I, X_I)\} \in \mathbb{R}_{++}^{2|N|+1}$  that are not generic is the union of finitely many hyperplanes and therefore has Lebesgue measure zero.

<sup>5</sup>This assumption is also made without much loss of generality, as the set of functions from  $\mathbb{R}^2$  to  $\mathbb{R}$  for which the outputs coincide on a finite set of distinct inputs forms a measure-zero set in the space of all functions from  $\mathbb{R}^2$  to  $\mathbb{R}$ .

**Axiom 3** (Rationality). For any  $I \in \mathcal{W}$ , any  $I' \in \phi(I)$ , and any  $I'' \in \mathcal{W}$ ,

$$I'' \notin \phi(I) \iff G(I'') < G(I').$$

These axioms are natural and capture the economic forces that give rise to the subgame perfect equilibria of the game in Section 3.2. Axiom 1 requires  $\phi$  to map any initial winning coalition to a non-trivial ruling coalition. Axiom 2 requires any ruling coalition selected by  $\phi$  to be a winning coalition. Axiom 3 imposes payoff-based selection: if  $I' \in \phi(I)$ , then no winning coalition  $I''$  with strictly lower  $G(\cdot)$  can be selected, and conversely any winning coalition with strictly higher  $G(\cdot)$  must be selected. Proposition 1 establishes that these axioms pin down a unique mapping under Assumptions 1–2, and that the correspondence is single-valued under Assumptions 1–3.

**Proposition 1.** 1. (Existence) Under Assumptions 1–2, the unique correspondence that satisfies Axioms 1–3 is

$$\phi(I) = \arg \max_{W \in \mathcal{W}} G(W).$$

2. (Uniqueness) Under Assumptions 1–3, the correspondence  $\phi$  is single-valued.

Proposition 1 is straightforward. It shows that the ruling coalition is a winning coalition that maximizes plunder, i.e., it maximizes  $G(W)$  among all  $W \in \mathcal{W}$ . Under Assumption 3, this coalition is unique.

## 3.2 The non-cooperative extensive game

We next define the extensive-form complete-information game

$$\Gamma = (N, I_0, p(\cdot), x(\cdot), \{U_i(\cdot)\}_{i \in N}, \beta),$$

where  $N$  is the set of players,  $I_0$  is the initial winning coalition,  $p(\cdot)$  and  $x(\cdot)$  are the power and resource mappings,  $\{U_i(\cdot)\}_{i \in N}$  are the payoff functions satisfying Assumption 1 and Assumption 2, and  $\beta \in (1/2, 1]$  is the supermajority requirement. The game starts with the initial winning coalition  $I_0 \in \mathcal{W}$ , and the steps are as follows:

1. Nature randomly picks an agenda setter  $a_q$  from the initial winning coalition, with  $q = 1$ , where  $q \in \{1, \dots, |I_0|\}$  denotes the round of agenda setting and voting.
2. The agenda setter  $a_q$  proposes a coalition  $I_q \subseteq N$ . If  $P_{I_q} < \beta P_N$ , then the game proceeds to Step 4. Otherwise, Nature chooses an order of votes and the game proceeds to Step 3.

3. The voting process begins. The coalition  $I_q$  forms if and only if the proposal of  $a_q$  is accepted by *all* players in  $I_q$ . In this case,  $I_q$  becomes the ruling coalition, and each player  $i \in N$  receives payoff  $U_i(I_q) = x_i + w_i(I_q)$ . Otherwise, following the first rejection of the proposal, the game proceeds to Step 4.
4. If  $q < |I_0|$ , Nature randomly picks a *new* agenda setter  $a_{q+1} \in I_0 \setminus \{a_1, a_2, \dots, a_q\}$  and the game returns to Step 2. If  $q = |I_0|$ , then  $I_0$  becomes the ruling coalition and each player  $i \in N$  receives payoff  $U_i(I_0) = x_i + w_i(I_0)$ .

The solution concept is subgame perfect equilibrium (SPE). The extensive-form game specifies players' strategies in any such equilibrium. A pure strategy of any player  $i \in I_0$  is a pair of functions  $\sigma_i(h) = (v_i(h, \mathcal{P}), \mathcal{P}_i(h))$  specifying her behavior at each decision node  $h$ : the function  $v_i(h, \mathcal{P})$  specifies player  $i$ 's vote (either 'Yes' or 'No') in any history  $h$  where Nature selects her to vote on a proposal  $\mathcal{P}$ , and  $\mathcal{P}_i(h)$  specifies the coalition that player  $i \in I_0$  proposes if selected by Nature as the agenda setter in history  $h$ . According to the extensive-form game, if  $i \in N \setminus I_0$ , player  $i$  cannot propose a coalition throughout the game.<sup>6</sup> Thus, the strategy of any  $i \in N \setminus I_0$  is the voting function  $v_i(h, \mathcal{P})$ , which assigns either 'Yes' or 'No' to any proposed ruling coalition  $\mathcal{P}$  containing  $i$  in any history  $h$  where Nature selects her to vote on  $\mathcal{P}$ .

We now establish existence and uniqueness of the ruling coalition in the non-cooperative coalition formation game, a preliminary result that prepares our analysis of the equilibrium's resilience to exogenous shocks. We also show that the SPE outcome of the coalition formation game coincides with the ruling coalition characterized by the axiomatic approach in Section 3.1.

- Proposition 2.** *1. (Existence) Suppose that Assumptions 1–2 hold and that  $\phi(I_0)$  satisfies Axioms 1–3. Then, for any  $I \in \phi(I_0)$ , there exists a pure-strategy SPE  $\sigma_I$  that produces  $I$  as the ruling coalition. In this SPE, each player  $i \in N$  receives payoff  $U_i(I) = x_i + w_i(I)$ .*
- 2. (Uniqueness) Suppose that Assumptions 1–3 hold, that  $\phi(I_0)$  satisfies Axioms 1–3, and that  $\phi(I_0) = \{I\}$ . Then, in any SPE,  $I$  is the ruling coalition. In particular, in any SPE, each player  $i \in N$  receives payoff  $U_i(I) = x_i + w_i(I)$ .*

The intuition is straightforward given Assumptions 1–3 and the axiomatic characterization in Proposition 1, where  $\phi(I_0) = \arg \max_{W \in \mathcal{W}} G(W)$  for any  $I_0 \in \mathcal{W}$ . Any ruling coalition  $I$  identified by the axiomatic analysis (i.e.,  $I \in \phi(I_0)$ ) can be supported by an SPE in which every agenda setter from  $I_0$  proposes  $I$ , and every voter from  $I_0$  accepts  $I$

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<sup>6</sup>All results continue to hold if the game is modified so that all players can be both voters and proposers.

and rejects any other proposal. Under the supermajority rule  $\beta \in (1/2, 1]$ , any coalition  $I'$  proposed before  $I$  must include at least one player from  $I$ . Since coalitions form under unanimity, the proposed strategy prevents any such  $I'$  from becoming the ruling coalition, ensuring that  $I$  forms.<sup>7</sup> Moreover, in the axiomatic analysis, Assumption 3 implies that the correspondence  $\phi : \mathcal{W} \rightarrow 2^N$  is single-valued, so any SPE yields the same ruling coalition.

**Remark: potential ruling coalitions.** We then define “potential ruling coalitions,” which is a simple property of the equilibrium ruling coalition and a useful tool to understand coalitional resilience.

**Definition 2.** *For any power and resource mappings  $p(\cdot)$  and  $x(\cdot)$ , define the set of potential ruling coalitions as*

$$Z := \{I \in \mathcal{W} \mid \nexists I' \in \mathcal{W} \text{ such that } P_{I'} > P_I \text{ and } X_{I'} < X_I\}. \quad (3.1)$$

Figure 2 illustrates the potential ruling coalitions (blue dots). A winning coalition is a potential ruling coalition if and only if no other winning coalition dominates it in both power and resources. The ruling coalition in Proposition 2 must therefore be a potential ruling coalition—a simple observation that will prove useful in what follows.

**Remark: Robustness without Assumption 2.** While Assumption 2 pins down a unique ruling coalition, our subsequent resilience analysis does not require it. We only need to assume a consistency of preferences on the set of potential ruling coalitions  $Z$ ; in this case, a ruling coalition is already well defined (Figure 3) and we can proceed to characterize its resilience. As we will discuss below, analysis under Assumption 2 is actually informative on the general case with preference heterogeneity across members: preference heterogeneity tends to reduce external resilience, so results under Assumption 2 provide an upper bound on the external resilience of a ruling coalition whose members disagree about what constitutes a good alternative.

## 4 Main analysis on coalitional resilience

This section studies the resilience of the ruling coalition, which is the central part of the paper. Our analysis proceeds in three steps. We begin by characterizing the ruling coalition, which allows us to define “internal” and “external” resilience – robustness to

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<sup>7</sup>As to off-path equilibrium strategies, they are characterized by Equation 3 of the proof for Proposition 2(1), which is in the appendix.

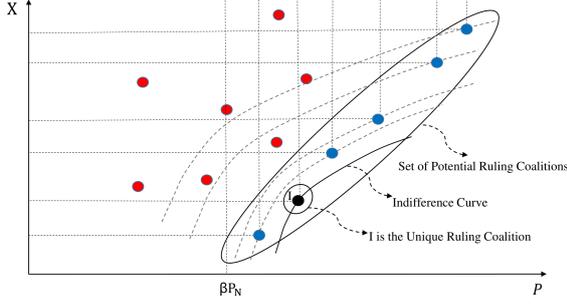


Figure 2:  $I$  is the unique ruling coalition of the game.

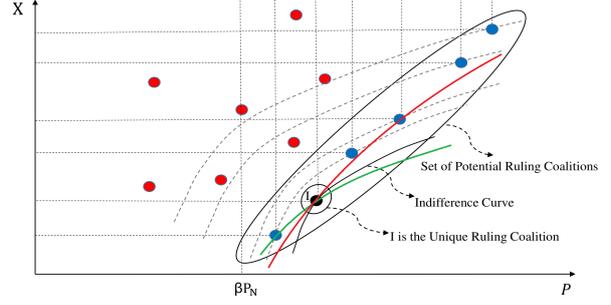


Figure 3:  $I$  is the unique ruling coalition under heterogeneous but consistent preferences over  $Z$  (each color represents one player's preferences over  $Z$ ).

changes in the power and resources of members versus outsiders (Proposition 3). The next two steps generate the central results of the paper. First, we show that the ruling coalition that is most externally resilient has a hierarchical structure (Proposition 4). Second, we uncover a potential trade-off between internal and external resilience, which depends on the “intensity” of power in plundering. We then comment on why our analysis of coalitional resilience may help understand the dynamics of the ruling coalition.

#### 4.1 Internal and external resilience

This section shows that it is necessary and sufficient for the ruling coalition to dominate two types of threats: sub-coalitions of the ruling coalition and alternative coalitions that include players outside the ruling coalition. This distinction reflects the central challenges from regime insiders and outsiders (Svolik (2012); Meng (2020); Paine (2021); Egorov and Sonin (2024)), enabling us to conceptualize two notions of resilience. To proceed, we first define the set of “best sub-coalitions.”

**Definition 3.** For any  $p(\cdot)$  and  $x(\cdot)$  and any subset of players  $I$ , define the set of best sub-coalitions of  $I$  as follows:

$$\mathcal{A}_I := \{A \subseteq I \mid A \neq \emptyset, \nexists A' \subseteq I \text{ such that } P_{A'} > P_A \text{ and } X_{A'} < X_A\}. \quad (4.1)$$

For any subset of players  $I \subseteq N$ ,  $\mathcal{A}_I$  includes the best subsets of  $I$ , i.e., those for which there does not exist another subset of  $I$  with both higher power and lower resources. Equation 4.1 is analogous to Equation 3.1 in Definition 3.1 for potential ruling coalitions, but restricts attention to sub-coalitions of the coalition in question. We can now prove Proposition 3, which characterizes the two types of threats to the ruling coalition. In particular, given a ruling coalition  $I$ , denote

$$A^{ins} \in (\mathcal{A}_I \setminus I) \cap \mathcal{W}$$

as a best sub-coalition of  $I$  that is also a winning coalition. A coalition  $A^{ins}$  is therefore an internal threat. Such a coalition  $A^{ins}$  can also form an alliance with outsiders to challenge the ruling coalition  $I$ , and these outsiders should also be best-subcoalitions of all outsiders. We denote a coalition of such outsiders as

$$A^{ext} \in \mathcal{A}_{N \setminus I}.$$

A ruling coalition only needs to defeat any internal threat  $A^{ins}$  and every alliance between any  $A^{ins}$  and most threatening outsiders  $A^{ext}$ . This intuition is established as follows.

**Proposition 3.** *Consider a game  $\Gamma = (I_0, p(\cdot), x(\cdot), \{U_i(\cdot)\}_{i \in N}, \beta)$  and suppose that Assumptions 1–3 hold. Then  $\phi(I_0) = \{I\}$  if and only if  $I \in \mathcal{W}$  and:*

- (i) *For all  $A^{ins} \in (\mathcal{A}_I \setminus \{I\}) \cap \mathcal{W}$ ,  $G(I) > G(A^{ins})$  (i.e., there is no profitable internal secession).*
- (ii) *For all  $A^{ext} \in \mathcal{A}_{N \setminus I}$  and for all  $A^{ins} \in \mathcal{A}_I$  with  $A^{ins} \cup A^{ext} \in \mathcal{W}$ ,  $G(I) > G(A^{ins} \cup A^{ext})$  (i.e., there is no profitable external secession).*

Proposition 3 shows that a necessary and sufficient condition for  $I$  to defeat all alternative winning coalitions is to dominate (i) all its nontrivial best sub-coalitions and (ii) all alliances between its best sub-coalitions with best sub-coalitions of outsiders. The following example illustrates the proposition.

Motivated by Condition (i) of Proposition 3, we now define the key object for our analysis of internal resilience. We can express Condition (i) in the  $(P, X)$  space as follows, using the indifference curve  $H_I(\cdot)$  over aggregate power  $P$  and aggregate resources  $X$  (Definition 1).

**Definition 4.** *For the ruling coalition  $I$ , the “internal safe area” is defined as*

$$\mathcal{S}_I^{\text{int}} := \{(P, X) \in \mathbb{R}_{++}^2 \mid X > H_I(P)\}. \quad (4.2)$$

*Then a ruling coalition  $I$  has the same internal resilience as a ruling coalition  $J$  if and only if  $\mathcal{S}_I^{\text{int}} = \mathcal{S}_J^{\text{int}}$ . A ruling coalition  $I$  is strictly (weakly) more internally resilient than a ruling coalition  $J$  if and only if  $\mathcal{S}_J^{\text{int}} \subsetneq \mathcal{S}_I^{\text{int}}$  (respectively,  $\mathcal{S}_J^{\text{int}} \subseteq \mathcal{S}_I^{\text{int}}$ ).*

For any coalition  $I$  to be the ruling coalition, all its best sub-coalitions must lie in  $\mathcal{S}_I^{\text{int}}$ . This guarantees that all members of  $I$  prefer  $I$  to any best sub-coalition of  $I$ , satisfying Condition (i) of Proposition 3. We write  $\mathcal{S}^{\text{int}}$  when there is no confusion. Figure 4 illustrates the internal safe area  $\mathcal{S}^{\text{int}}$  for a coalition  $I$ . This set plays a central role in our subsequent analysis of internal resilience. In particular, we will see that a

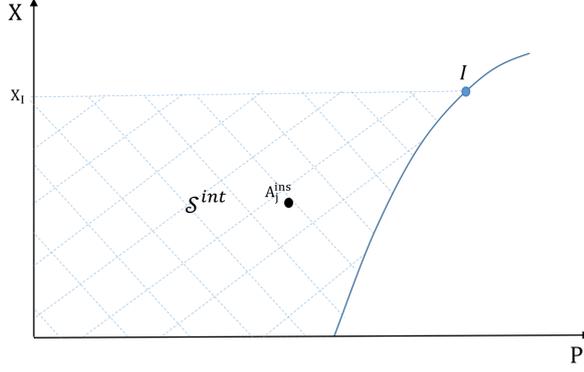


Figure 4: The internal safe area  $\mathcal{S}^{\text{int}}$  is the shaded region.

coalition  $I$  remains stable if, after an exchange of power and resources within  $I$ , all its best sub-coalitions remain inside the internal safe area  $\mathcal{S}^{\text{int}}$ .

We now turn to “external” threats. For any best sub-coalition of outsiders  $A^{\text{ext}}$  and any best sub-coalition of insiders  $A^{\text{ins}}$ , Condition (ii) of Proposition 3 requires  $G(I) > G(A^{\text{ins}} \cup A^{\text{ext}})$ , which is equivalent to  $X_{A^{\text{ins}} \cup A^{\text{ext}}} > H_I(P_{A^{\text{ins}} \cup A^{\text{ext}}})$ . Since aggregate power and resources are additive, this condition can be written as  $X_{A^{\text{ins}}} + X_{A^{\text{ext}}} > H_I(P_{A^{\text{ins}}} + P_{A^{\text{ext}}})$ , or

$$X_{A^{\text{ins}}} > H_I(P_{A^{\text{ins}}} + P_{A^{\text{ext}}}) - X_{A^{\text{ext}}}. \quad (4.3)$$

For any group of outsiders  $A^{\text{ext}}$  that satisfy Condition 4.3, the group of outsiders cannot threaten the ruling coalition. We need to make sure that Condition 4.3 is satisfied for every best sub-coalition of the ruling coalition. This motivates the following definition of the “external safe area” for a ruling coalition  $I$ .

**Definition 5.** Consider a ruling coalition  $I$  with its set of best sub-coalitions  $\mathcal{A}_I$ . For any  $A_j^{\text{ins}} \in \mathcal{A}_I$ , define

$$H_I^j(P) := H_I(P + P_{A_j^{\text{ins}}}) - X_{A_j^{\text{ins}}}, \quad (4.4)$$

The function 4.4 is obtained by shifting the indifference curve  $H_I(P)$  leftward by  $P_{A_j^{\text{ins}}}$  units and downward by  $X_{A_j^{\text{ins}}}$  units.

The area externally safe relative to  $A_j^{\text{ins}}$  is

$$\mathcal{S}_j^{\text{ext}} := \{(P, X) \in \mathbb{R}_{++}^2 \mid P < \beta P_N \text{ and } X > H_I^j(P)\}. \quad (4.5)$$

Define the external safe area for the ruling coalition  $I$  as

$$\mathcal{S}_I^{\text{ext}} := \bigcap_{A_j^{\text{ins}} \in \mathcal{A}_I} \mathcal{S}_j^{\text{ext}}. \quad (4.6)$$

A ruling coalition  $I$  has the same external resilience as a ruling coalition  $J$  if and only

if  $\mathcal{S}_I^{\text{ext}} = \mathcal{S}_J^{\text{ext}}$ . A ruling coalition  $I$  is strictly (weakly) more externally resilient than a ruling coalition  $J$  if and only if  $\mathcal{S}_J^{\text{ext}} \subsetneq \mathcal{S}_I^{\text{ext}}$  (respectively,  $\mathcal{S}_J^{\text{ext}} \subseteq \mathcal{S}_I^{\text{ext}}$ ).

What is the intuition behind Definition 5 and “external safety”? First, outsiders cannot have  $P^{\text{ext}} \geq \beta P_N$ . If they did, this outsider group would itself have supermajority power. Since  $P_I \geq \beta P_N$  is required for  $I$  to be a ruling (therefore necessarily winning) coalition, such an outsider group can form a winning coalition on its own, and  $I$  could not remain the ruling coalition. Hence  $P^{\text{ext}} < \beta P_N$  is a necessary condition for an outsider group to be “safe” for any ruling coalition  $I$ .

Second, fix a best sub-coalition of insiders  $A_j^{\text{ins}} \in \mathcal{A}_I$ . For any best sub-coalition of outsiders  $A^{\text{ext}}$  that lies exactly on the shifted indifference curve  $H_I^j(P)$ , insiders are indifferent between the current ruling coalition  $I$  and the alternative coalition  $A_j^{\text{ins}} \cup A^{\text{ext}}$ . If  $A^{\text{ext}}$  lies in the region  $\mathcal{S}_j^{\text{ext}}$ , then  $A_j^{\text{ins}} \cup A^{\text{ext}}$  is strictly worse than  $I$ , so outsiders with such  $(P, X)$  cannot combine with  $A_j^{\text{ins}}$  to form a profitable external deviation. In this sense,  $\mathcal{S}_j^{\text{ext}}$  is “externally safe” relative to  $A_j^{\text{ins}}$ . Taking the intersection over all  $A_j^{\text{ins}} \in \mathcal{A}_I$  yields the external safe area  $\mathcal{S}_I^{\text{ext}}$ , i.e., the set of outsider best sub-coalitions that are simultaneously safe against *any* insider best sub-coalition.

Proposition 3 yields an additional insight: a ruling coalition must maintain a sufficiently high power-to-resource ratio relative to any relevant alternative, as captured by both the internal and external safe areas.<sup>8</sup> This, for instance, offers a rationale for the voluntary destruction of resources by a ruling coalition when faced with a threatening alternative.

**Remark: Heterogeneity of preferences and resilience.** How does preference heterogeneity within the ruling coalition affect external resilience once Assumption 2 is relaxed? Since the external safe area is defined as the region above the envelope of the boundaries induced by insiders’ best sub-coalitions, introducing an insider whose preferences differ from others can only add an additional boundary, which can only shift the envelope weakly upward. Hence, the external safe area can only weakly shrink, i.e., external resilience weakly decreases. The same logic applies to the internal safe area.

Now that we have provided a precise characterization of internal and external safe areas, we are ready to study internal and external resilience of a ruling coalition.

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<sup>8</sup>Example 4 in the appendix demonstrates that neither the most powerful player nor the one with the fewest resources is necessarily included in the ruling coalition.

## 4.2 Which ruling coalitions are more externally resilient?

Consider a ruling coalition  $I$  and suppose that there are two players  $i, j \in I$ , with  $p_i > p_j$  and  $x_i < x_j$ . Holding the power and resources of all other players fixed, transfer either (i) a portion of player  $i$ 's power to player  $j$ , with  $0 < \Delta p \leq \frac{p_i - p_j}{2}$ , or (ii) a portion of player  $j$ 's resources to player  $i$ , with  $0 < \Delta x \leq \frac{x_j - x_i}{2}$  (Figure 5). The following proposition establishes that such an equalizing transfer between members (weakly) reduces the threat posed by the stronger or poorer member, and therefore (weakly) increases the coalition's external resilience. This result is general and does not depend on the precise specification of the plundering function  $G(\cdot)$ , i.e., on the shape of indifference curves.

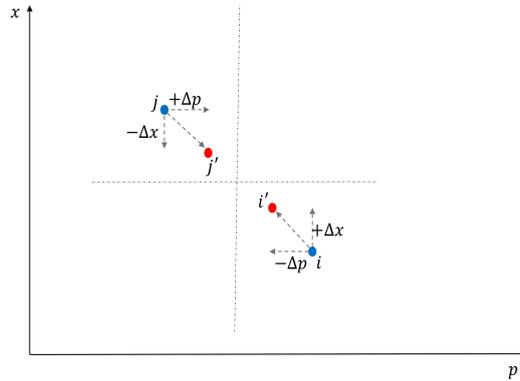


Figure 5: Exchange of powers and resources between player  $i$  and player  $j$ —from blue to red.

**Proposition 4.** *Suppose  $I$  is the unique ruling coalition of the game  $\Gamma$ , and there exist  $i, j \in I$  with  $p_i > p_j$  and  $x_i < x_j$ . Holding fixed the powers and resources of players in  $I \setminus \{i, j\}$ , consider the modified coalition  $(I \setminus \{i, j\}) \cup \{i', j'\}$  where*

$$p_{i'} = p_i - \Delta p, \quad x_{i'} = x_i + \Delta x, \quad p_{j'} = p_j + \Delta p, \quad x_{j'} = x_j - \Delta x,$$

for any  $0 < \Delta p \leq \frac{p_i - p_j}{2}$  and  $0 < \Delta x \leq \frac{x_j - x_i}{2}$ . Then the external resilience of  $(I \setminus \{i, j\}) \cup \{i', j'\}$  is weakly higher than the external resilience of  $I$ .

Proposition 4 is the first key result on coalitional resilience. The proof is in Appendix A and proceeds in three steps.

*Step 1.* We identify two effects of the exchange from  $\{i, j\}$  to  $\{i', j'\}$ : (i) some best sub-coalitions of  $I$  gain resources and lose power, making them less threatening; and (ii) the set of best sub-coalitions may itself change—new ones may emerge and old ones may cease to qualify.

*Step 2.* We show that neither effect reduces external resilience. Effect (i) is straightforward. For effect (ii), observe that before the exchange, some sub-coalition must have been (weakly) more threatening than any newly emerging best sub-coalition—that is, it

had weakly higher power and weakly lower resources. This follows by contradiction: if no such sub-coalition existed, the emerging best sub-coalition would already have been best before the exchange. Hence no newly emerging best sub-coalition is more threatening than a previously best one, which cannot reduce external resilience. A previously best sub-coalition that ceases to qualify also cannot reduce external resilience (Proposition 3).

*Step 3.* We show that these changes do not generate a profitable internal secession. Indeed, if a ruling coalition  $I$  already withstands internal secession before the exchange (Condition (i) of Proposition 3), internal secession cannot be triggered by replacing some best sub-coalitions with a strictly less threatening set.

**A new justification for hierarchy.** As an important implication of Proposition 4, iterating the exchanges of power and resources depicted in Figure 5 yields “conditional equality” (or “conditional proportionality”) within the ruling coalition. Coalition members are partitioned into ranks: within each rank, members have identical power and resources, and across ranks, power and resources are proportional, with the highest rank holding the most power and resources, followed by the second rank, and so on. At each step, external resilience weakly increases. Consequently, the resulting allocation has weakly higher external resilience than the initial allocation. In this sense, iterated exchanges produce a “hierarchy” (Figure 6). Our analysis thus offers a new perspective on why the most resilient plundering coalitions tend to exhibit hierarchical organization with well-defined ranks. Examples include stable oligarchies, Weberian bureaucracies, and armies.

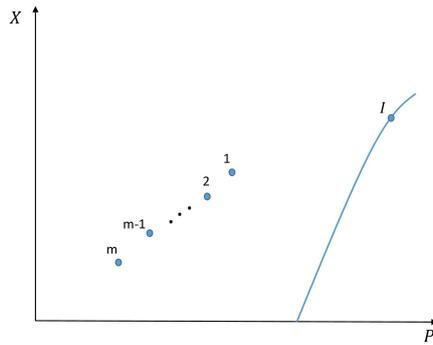


Figure 6: A coalition consisting of  $m$  ranks. Each blue dot represents a rank of players with identical power and resources and ranks are totally ordered by power  $P$  and resources  $X$ .

It is important to note that comparing the external resilience of different “hierarchies”—i.e., different proportional configurations of power and resources within the ruling coalition—generally requires additional structure on the plundering function  $G(\cdot)$ . For instance, fix the total power and resources of a two-player ruling coalition  $I$  and consider two internal configurations,  $\{i, j\}$  and  $\{i', j'\}$ , such that  $P_I = p_i + p_j = p_{i'} + p_{j'}$  and  $X_I = x_i + x_j = x_{i'} + x_{j'}$  (Figure 7). Then there is no general argument that ranks which configura-

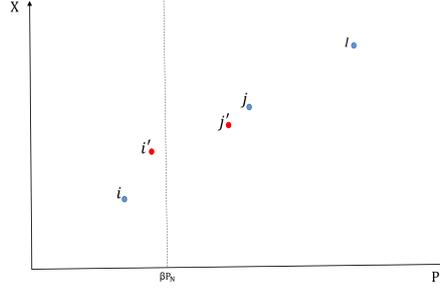


Figure 7: Ambiguity in comparing external resilience of proportional internal distributions.

tion yields higher external resilience without further restrictions on  $G(\cdot)$  (e.g., beyond concavity of indifference curves).

**Dynamic implications.** It is natural to ask what happens after outsiders are plundered. Plundering depletes outsiders’ resources and raises their power-to-resource ratios, potentially making them more attractive coalition members in the future and, in turn, destabilizing the very coalition that plundered them.

Our static game and resilience analysis can be interpreted as a benchmark for a repeated *stationary-bandit* environment: for each “season,” the ruling coalition extracts and consumes the extracted resources, while outsiders rebuild resources through (agricultural) production before the next season. Each season then starts from a recovered distribution  $x(\cdot)$  and the same coalition-formation and extraction problem is played again. In such a setting, extraction can persist period after period if the ruling coalition consume all resources and preserves outsiders’ productive capacity (e.g., does not burn the land or kill the farmers), unless an exogenous shock disrupts the process. In this sense, our resilience analysis provides a simple benchmark for understanding how extractive coalitions evolve over time in the presence of shocks.

**Preference heterogeneity and hierarchy** Importantly, the logic underlying Proposition 4 extends to heterogeneous preferences within the ruling coalition. We first set aside a degenerate case: if some insider strictly prefers a different ruling coalition, resilience is trivially zero under any external shock. Absent this, Proposition 4 extends under a weaker version of Assumption 2 that accommodates preference heterogeneity. The key force in the proof remains unchanged: the exchange shifts certain best sub-coalitions up and left in  $(P, X)$ -space, which pushes down the relevant boundary curves in Equation 4.4 that jointly defines the external safe area (Definition 5). Under Assumption 1, for any insider (regardless of the curvature of her indifference curves), a sub-coalition that moves up/left becomes weakly less attractive after the exchange. Hence, the associated boundary curve for that insider shifts weakly downward; aggregating across insiders, this

weakly expands the external safe area. Therefore, the exchange in Figure 5 enlarges the external safe area in the same direction as in the benchmark case of homogeneous preferences under Assumption 2.

In Proposition 4, the exchange of power and resources *weakly* increases the external resilience of the ruling coalition. The next sections characterize conditions under which the exchange *strictly* increases external resilience, as well as conditions under which external resilience remains unchanged. In particular, we highlight the role of the shape of indifference curves—convex versus concave—which we interpret as capturing the strength of property-rights protection.

### 4.3 Power-intensive and power-light plundering

Concave and convex indifference curves capture a fundamental difference in how the marginal value of power varies with a coalition’s power. For an indifference curve  $H_I(P)$  that goes through a ruling coalition  $I$ , the marginal value of power is:

$$\frac{d}{dP}H_I(P).$$

To understand this, consider a small increase in a coalition’s power. To keep insiders indifferent, by how much must the coalition’s resources increase? Recall that insiders dislike resources held *inside* the coalition, since these resources are protected from plunder. If  $\frac{d}{dP}H_I(P)$  is large, a small increase in power can offset a large increase in internal resource that cannot be plundered, indicating a high marginal value of power.

**Definition 6.** 1. Under concave indifference curves ( $\frac{d^2}{dP^2}H_I(P) < 0$ ), power displays decreasing marginal return, or **decreasing return to power**.

2. Under convex indifference curves ( $\frac{d^2}{dP^2}H_I(P) > 0$ ), power displays increasing marginal return, or **increasing return to power**.

Under concave indifference curves ( $\frac{d^2}{dP^2}H_I(P) < 0$ ), the marginal value of power decreases as power increases. Consequently, a ruling coalition with many members, hence a high aggregate power  $P$ , displays a very low marginal value of power  $\frac{d}{dP}H_I(P)$ . This induces the ruling coalition to have relatively few members, so that the coalition can also keep more resources outside and extractable.<sup>9</sup>

By contrast, under convex indifference curves, the marginal value of additional power increases with the power of a coalition. This induces the formation of a more “inclusive”

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<sup>9</sup>This preference has an analogy in standard consumer theory: although a consumer prefers more of a good, diminishing marginal utility implies that extremely large amounts of the same good are not valuable at the margin.

ruling coalition with high aggregate power. For instance, when institutions constrain plundering, inclusive ruling coalitions may have an advantage because greater aggregate power helps overcome these constraints.<sup>10</sup>

The following proposition shows that, under convex indifference curves or increasing return to power, the internal configuration of power and resources does not affect external resilience.

**Proposition 5** (Power-intensive plundering and invariance of external resilience). *Suppose that preferences over coalitions  $(P, X)$  have strictly convex indifference curves. For any ruling coalition  $I$  and any  $A \in \mathcal{A}_I \setminus \{I\}$ , we have*

$$\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_A^{\text{ext}}.$$

Hence,

$$\mathcal{S}^{\text{ext}} := \bigcap_{A \in \mathcal{A}_I} \mathcal{S}_A^{\text{ext}} = \mathcal{S}_I^{\text{ext}}.$$

Therefore, any exchange of power and resources within  $I$  that preserves internal stability leaves the external resilience of  $I$  unchanged.

Proposition 5 is proved in Appendix A. Fix a ruling coalition  $I$  and  $A^{\text{ins}} \in \mathcal{A}_I \setminus \{I\}$ , a nontrivial best sub-coalition of  $I$  (an insider sub-coalition). Under strictly convex indifference curves (increasing return to power), insiders strictly prefer larger ruling coalitions. Hence, for any best sub-coalition of outsiders  $B \in \mathcal{A}_{N \setminus I}$ , insiders in  $A^{\text{ins}}$  prefer  $B \cup I$  to  $B \cup A^{\text{ins}}$ . Any profitable and feasible external deviation of the form  $B \cup A^{\text{ins}}$  is therefore (weakly) dominated by the deviation  $B \cup I$ . Thus, the set of outsider coalitions that can induce secession with  $A^{\text{ins}}$  is contained in the set that can induce secession with  $I$ , which implies that  $\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_{A^{\text{ins}}}^{\text{ext}}$ . Since  $I$  is itself included among the insider best sub-coalitions, we obtain

$$\mathcal{S}^{\text{ext}} := \bigcap_{A_i^{\text{ins}} \in \mathcal{A}_I} \mathcal{S}_i^{\text{ext}} = \mathcal{S}_I^{\text{ext}}.$$

Therefore, as long as internal exchanges of power and resources do not trigger internal secession, they leave  $\mathcal{S}^{\text{ext}}$ —and hence external resilience—unchanged.

Next, we show that when the plundering technology displays sufficient decreasing return to power (i.e., indifference curves are sufficiently concave), external resilience *strictly* increases as we transition to a hierarchy with well-defined ranks by iterating the exchanges in Figure 5 within the ruling coalition.

To measure concavity, we focus on a CES family of plundering functions  $\{G_\rho\}_{\rho \neq 0}$ .

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<sup>10</sup>Example 3 in the appendix provides a more detailed discussion of environments that feature increasing return and decreasing return to power.

Fix a ruling coalition  $I$  with aggregate power and resources  $(P_I, X_I)$ . For  $\alpha \in (0, 1)$  and  $\rho \neq 0$ , define

$$G_\rho(P, X) := \left[ \alpha \left( \frac{P}{P_I} \right)^\rho + (1 - \alpha) \left( \frac{\bar{X} - X}{\bar{X} - X_I} \right)^\rho \right]^{1/\rho},$$

where  $\bar{X}$  is assumed to be sufficiently large (with  $\bar{X} > X_I$ ) and  $(P, X) \in \mathbb{R}_+ \times (0, \bar{X})$ . For  $\rho \leq 1$ ,  $G_\rho$  is concave in  $(P, X)$  (Figure 8), corresponding to increasingly power-light plundering as  $\rho$  decreases.<sup>11</sup>

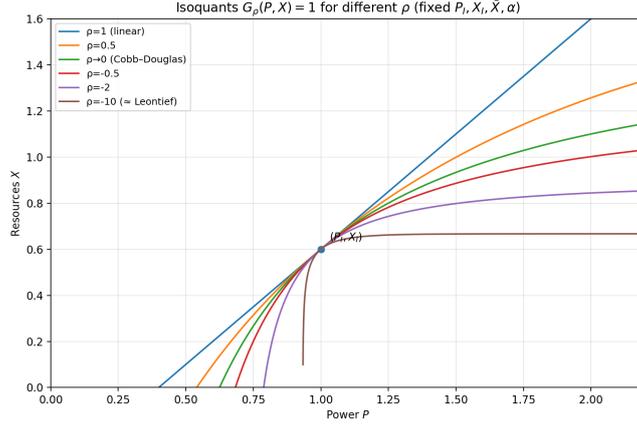


Figure 8: Isoquants of the CES plundering technology  $G_\rho(P, X)$  for different values of  $\rho$ , normalized to pass through  $(P_I, X_I)$ . Lower  $\rho$  implies a more concave/ Leontief-like shape.

The next proposition shows that for any initial distribution of power and resources within the ruling coalition, there exists an indifference curve passing through  $I$  that is sufficiently concave—i.e., a sufficiently small  $\rho$  in the CES family  $\{G_\rho\}$ —such that iterating the exchange in Figure 5 until the coalition becomes hierarchical *strictly* increases external resilience.<sup>12</sup>

**Proposition 6** (Decreasing return to power and strict gains from hierarchy). *Suppose that  $I$  is a ruling coalition and preferences over coalitions  $(P, X)$  are given by  $G_\rho(\cdot)$  for some  $\rho < 1$ . Further suppose that  $I'$  is the allocation obtained from  $I$  by iterating the bilateral exchanges in Figure 5 until the coalition becomes a hierarchy with well-defined ranks. Then there exists  $\bar{\rho} < 1$  such that if  $\rho \leq \bar{\rho}$ ,*

$$\mathcal{S}_I^{\text{ext}} \subsetneq \mathcal{S}_{I'}^{\text{ext}},$$

*so reaching a hierarchical allocation under strictly increases external resilience under sufficiently decreasing return to power.*

<sup>11</sup>The elasticity of substitution between power  $P$  and the resource-loss slack  $\bar{X} - X$  is  $\sigma = \frac{1}{1-\rho}$ . Thus, lower  $\rho$  implies lower  $\sigma$ , and the Leontief limit obtains as  $\rho \rightarrow -\infty$ .

<sup>12</sup>The logic is not specific to the CES class. We use this family only because it provides a transparent notion of “sufficient concavity” (via  $\rho$ ) and allows for a rigorous and tractable argument.

The proof is in Appendix A. Consider the sequence of bilateral exchanges within  $I$  described in Figure 5, and suppose these exchanges are repeated until there remain two insiders  $i, j \in I$  such that  $p_i > p_j$  and  $x_i < x_j$  and the exchange is still applicable. If the external safe area has already expanded at some earlier step, the result follows because Proposition 4 implies that the final exchange cannot shrink the external safe area. Thus, assume that external resilience has remained constant up to this point.

Denote the sub-coalition  $A_1^{\text{ins}} := I \setminus \{j\}$  and define the shifted indifference curve through this sub-coalition as

$$H_I^1(P) := H_I(P + P_{A_1^{\text{ins}}}) - X_{A_1^{\text{ins}}}.$$

It is straightforward to show that  $A_1^{\text{ins}}$  is a best insider sub-coalition of  $I$ . After the exchange between  $i$  and  $j$ ,  $A_1^{\text{ins}}$  moves left and up in  $(P, X)$ -space, e.g., to  $A_2^{\text{ins}}$  as in Figure 10. Under Assumption 1, for *any* indifference curve, a move from  $A_1^{\text{ins}}$  to  $A_2^{\text{ins}}$  strictly enlarges the corresponding external safe area, i.e.,

$$\mathcal{S}_1^{\text{ext}} \subsetneq \mathcal{S}_2^{\text{ext}}.$$

<sup>13</sup> Moreover, no sub-coalition that includes  $j$  but excludes  $i$  can be a best insider sub-coalition after the exchange. Otherwise, replacing  $j$  with  $i$  would yield a sub-coalition with weakly higher power and weakly lower resources (with at least one strict inequality), which is a contradiction.

Therefore, the external safe area can expand strictly at this step only if  $H_I^1(P)$  is binding in the construction of  $\mathcal{S}_I^{\text{ext}}$ , i.e., only if  $H_I^1(P)$  uniquely determines the upper envelope

$$\max_{A_i^{\text{ins}} \in \mathcal{A}_I} H_I^i(P)$$

for some  $P \in (0, \beta P_N)$ . The proof shows that for any given initial allocation of power and resources within the ruling coalition, there exists a sufficiently concave indifference curve  $H_I$  (equivalently, a sufficiently small  $\rho$ ) such that the boundary  $H_I^1(P)$  is *uniquely* binding at some  $P \in (0, \beta P_N)$ ; that is,  $H_I^1(P)$  uniquely attains  $\max_{A_i^{\text{ins}} \in \mathcal{A}_I} H_I^i(P)$  at that  $P$ .

To shed further light on the geometry, consider the extreme case  $\rho \rightarrow -\infty$ , where the plundering technology becomes Leontief and the indifference curve in  $(P, X)$ -space is an inverse- $L$  with a kink at  $(P_I, X_I)$  (Figure 9). In this limit, for any initial allocation of power and resources within  $I$  and any associated collection of best sub-coalitions  $\mathcal{A}_I$ , the boundary induced by each best sub-coalition (e.g.,  $A$  and  $B$  in Figure 9) uniquely

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<sup>13</sup>This is shown in the proof of Proposition 4.

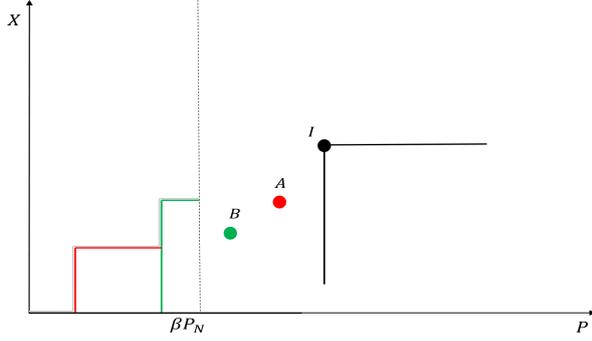


Figure 9: Under sufficiently concave (Leontief-like in limit) indifference curves, the boundary corresponding to a best sub-coalition is uniquely binding on the upper envelope that defines the external safe area.

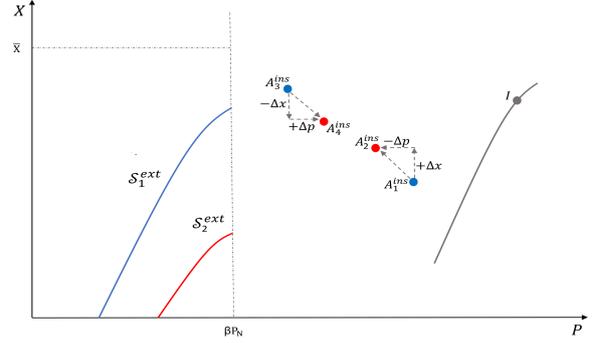


Figure 10: Under concave indifference curves, the exchange in Figure 5 expands the external safe area for best sub-coalitions that contain  $i$  (the member with higher power but lower resources than  $j$ ).

determines the upper envelope

$$\max_{A_I^{ins} \in \mathcal{A}_I} H_I^i(P)$$

on a nonempty interval of  $P \in (0, \beta P_N)$ . For instance, as Figure 9 illustrates, the boundary of the external safe area of  $I$  (the grey envelope) is piecewise determined in an ordered way: the right segment (green) is pinned down by the least-powerful best sub-coalition  $B$ , the middle segment (red) by the next-most powerful best sub-coalition  $A$ , and the left segment (black) by  $I$  itself (viewed as a best sub-coalition of itself). Therefore, when  $\rho$  is sufficiently low—i.e., indifference curves are sufficiently concave—the exchange between  $i$  and  $j$  strictly expands the external safe area, and iterating exchanges until the coalition becomes hierarchical strictly increases external resilience under power-light plundering.

**Remark 1** (Weak property rights and the emergence of hierarchy). *Our analysis so far highlights a central insight: the resilience advantage of hierarchy is stronger when property rights are weak than when they are strong. Proposition 4 shows that moving the ruling coalition toward a hierarchical allocation never decreases external resilience, under any plundering function  $G(\cdot, \cdot)$  satisfying Assumptions 1–3. We then contrast power-intensive and power-light plundering environments, interpreting convex versus concave indifference curves as capturing the strength of constraints on extraction (e.g., property-rights protection). As a hierarchy emerges, external resilience is unchanged under strong property-rights protection (power-intensive plundering; Proposition 5), but it strictly increases when property rights are sufficiently weak (power-light plundering; Proposition 6). Taken together, these results suggest that when constraints on plundering are weak, organizing the ruling coalition as a hierarchy can deliver a strict resilience gain against outsider threats, whereas this benefit is absent when property rights are well protected.*

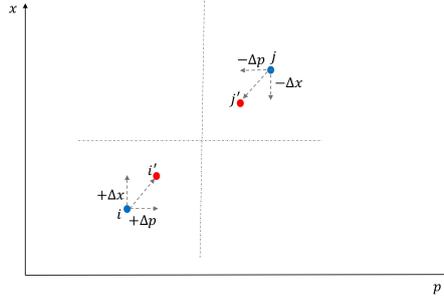


Figure 11: Exchange of power and resources between player  $i$  and player  $j$ —from blue to red.

### 4.3.1 Coalitions of absolutely equal members

It is important to note that the analysis above does not imply that a ruling coalition is, in general, most externally resilient when its members are absolutely equal. Without further restrictions on the plundering function and on the joint distribution of power and resources within the ruling coalition, no general ranking is possible. That said, an absolutely equal coalition does maximize external resilience in a particular case: when the most powerful member in the ruling coalition also has the lowest resources, the second most powerful has the second lowest resources, and so on. In that case, it is straightforward to implement a sequence of exchanges of the type in Figure 5 that converges to absolute equality.<sup>14</sup> The following examples illustrate a more general case in which the exchange of power and resources depicted in Figure 11 within the ruling coalition also increases external resilience. Together with Proposition 4, this generates a broad range of single-class coalitions.

**Example 1.** Consider a ruling coalition  $I = \{i, j\}$  with  $p_i < p_j$  and  $x_i < x_j$ . Let  $H_I(\cdot)$  denote the (concave) indifference curve through  $I$ . Suppose player  $i$  is more threatening than  $j$ , i.e.,  $\mathcal{S}_i^{\text{ext}} \subset \mathcal{S}_j^{\text{ext}}$ . If we perform the exchange in Figure 11 between  $i$  and  $j$  and obtain  $\mathcal{S}_{i'}^{\text{ext}} \subset \mathcal{S}_{j'}^{\text{ext}}$  (the red region covers the blue region in Figure 12), then the external safe area of the ruling coalition expands under this exchange.

*Intuitively, this occurs because player  $i$  is much more threatening than player  $j$  given  $H_I(\cdot)$ . After the exchange, taking resources from player  $j$  may make her more threatening (moving from  $j$  to  $j'$ ), but this effect is dominated by the reduction in the threat posed by player  $i$  when she becomes better endowed (moving from  $i$  to  $i'$ ). This force is strongest under power-light plundering (high concavity), where low-resource insiders are especially threatening because they have high power-to-resource ratios.*

**Remark 2.** Although we do not provide a general ranking here, Example 1 suggests an important intuition: when property-rights protection is extremely weak and there exists a

<sup>14</sup>More precisely, start by exchanging between the two most powerful players until they become equal in both power and resources; then continue with the third most powerful player until the top three become equal; and so forth.

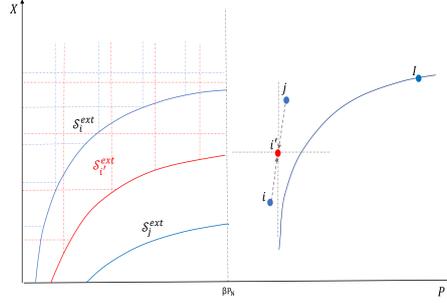


Figure 12: The increases in the external resilience due to an exchange of powers and resources as in Figure 12.

*poor but sufficiently powerful group within the ruling coalition, a single-class coalition can be particularly externally resilient. This perspective may help rationalize the egalitarian thrust of some communist revolutions—but specifically in settings where the revolutionary base is both economically disadvantaged and sufficiently powerful (e.g., urban proletarians in some historical contexts).*

## 4.4 Trade-off between internal and external resilience

We now link external and internal resilience by characterizing when a trade-off may arise between them.

### 4.4.1 Power-intensive plundering

We begin by showing that when plundering is power-intensive, a trade-off can arise between external and internal resilience as the plundering environment changes. Specifically, shifting toward more power-intensive plundering lowers external resilience but raises internal resilience. To formalize this comparative-static notion, we introduce a more general definition of the “intensity of plundering.” If the marginal rate of substitution between power and resources is strictly higher, then the marginal value of adding power to the ruling coalition is higher relative to the marginal cost of retaining internal resources that cannot be plundered. This corresponds to a more power-intensive (i.e., less power-light) plundering environment.

**Definition 7.** *The indifference curve  $H_I(\cdot)$  represents a more power-light plundering environment than  $H_I'(\cdot)$  if and only if for all  $P \in [\beta P_N, P_N]$ ,*

$$\frac{d}{dP}H_I(P) < \frac{d}{dP}H_I'(P),$$

*denoted by  $H \succ H'$ .*

**Proposition 7.** *Fix a ruling coalition  $I$ . Suppose the indifference curves  $H_I(\cdot)$  and  $H'_I(\cdot)$  are convex, and that internal and external shocks are independently distributed. Then  $H \succ H'$  if and only if the internal resilience of  $I$  under  $H$  is lower than under  $H'$  and its external resilience under  $H$  is higher than under  $H'$ .*

In a power-intensive plundering environment, the most threatening external deviations are those that include outsiders *and* all members of the ruling coalition, rather than those formed by outsiders together with a non-trivial insider subset. The reason is that power-intensive plundering (convex indifference curves) makes aggregate power increasingly valuable at the margin. Holding the outsider block fixed, expanding the insider component from a proper subset  $A^{\text{ins}} \subsetneq I$  to the full coalition  $I$  raises aggregate power, and under convexity this gain in power more than compensates for the fact that bringing in additional insiders also brings in protected internal resources that cannot be plundered. Hence, for any outsider best sub-coalition, deviations of the form  $A^{\text{ext}} \cup I$  are weakly preferred to deviations of the form  $A^{\text{ext}} \cup A^{\text{ins}}$ , implying that the external-safe constraint is effectively pinned down by deviations that involve  $I$  itself.

This has a direct implication for resilience. As the environment becomes more power-intensive, the propensity toward inclusiveness strengthens, so it becomes easier for outsiders to construct a profitable deviation precisely of the most threatening form  $A^{\text{ext}} \cup I$ . The external safe area therefore shrinks, and external resilience falls. At the same time, the same force raises internal resilience: because aggregate power is increasingly valuable, insiders are less tempted by breakaway sub-coalitions with lower power, so internal secession is less attractive. Thus, power-intensive plundering induces a trade-off: external resilience decreases while internal resilience increases (Figure 13).

**Remark 3.** *This implies that when property rights are relatively well protected—so plundering is more power-intensive—shifting toward lower plundering intensity can reduce the likelihood of an insider coup d’etat while increasing the risk of a popular uprising, and vice versa. In this sense, the ruling coalition faces a trade-off between stabilizing power-sharing among insiders and maintaining authoritarian control against outsiders.*

#### 4.4.2 Power-light plundering

As in the convex case, it is straightforward to show that moving toward less power-light plundering increases internal resilience when indifference curves are concave (Figure 14). However, how a change in plundering intensity affects external resilience is generally ambiguous in this region. Example 2 shows that a shift toward more power-intensive (i.e., less power-light) plundering can yield either higher or lower external resilience, depending on the nature of external perturbations.

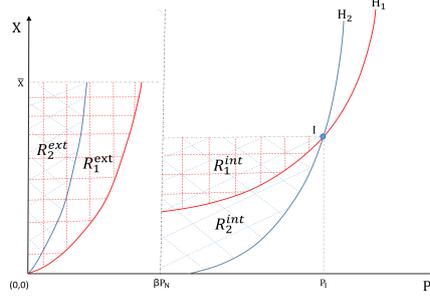


Figure 13: Trade-off between internal and external resilience under convex indifference curves.

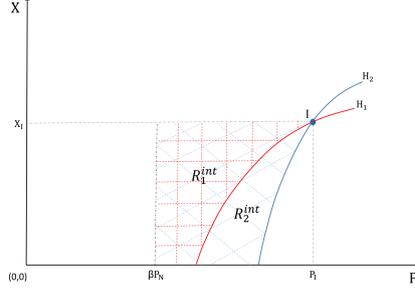


Figure 14: The increasing effect of a weaker plundering on internal resilience under concave indifference curves.

**Example 2.** Consider a ruling coalition  $I$  and a plundering technology represented by the indifference curve  $H_1$  passing through  $I$ . Suppose the plundering environment shifts toward more power-intensive plundering, moving from  $H_1$  to  $H_2$ . Suppose further that there is a best insider sub-coalition  $A_1^{\text{ins}} \in \mathcal{A}_I$  that is realized with probability one. The effect of the shift from  $H_1$  to  $H_2$  on external resilience is ambiguous (Figure 15): depending on the configuration of external perturbations, the external safe area associated with  $A_1^{\text{ins}}$  may expand or shrink.

For instance, consider two distributions over external shocks,  $Pr_1^{\text{ext}}(\cdot, \cdot)$  and  $Pr_2^{\text{ext}}(\cdot, \cdot)$ , where  $Pr_1^{\text{ext}}$  assigns relatively more probability mass to the outsider best sub-coalition  $A_2^{\text{ext}}$  than to  $A_1^{\text{ext}}$  (compared to  $Pr_2^{\text{ext}}$ ), as illustrated in Figure 15. Since internal and external shocks are independent, under  $Pr_1^{\text{ext}}$  the coalition  $A_2^{\text{ext}} \cup A_1^{\text{ins}}$  is more likely to arise than  $A_1^{\text{ext}} \cup A_1^{\text{ins}}$ . Suppose that moving from  $H_1$  to  $H_2$  makes  $A_1^{\text{ext}} \cup A_1^{\text{ins}}$  newly preferred to  $I$  (while it is not preferred under  $H_1$ ). Then external resilience falls under  $Pr_1^{\text{ext}}$  relative to  $Pr_2^{\text{ext}}$ , because the newly dangerous deviation receives more probability weight.

Alternatively, suppose  $Pr_2^{\text{ext}}$  assigns relatively more probability mass to  $A_1^{\text{ext}}$  than to  $A_2^{\text{ext}}$  (compared to  $Pr_1^{\text{ext}}$ ), so that  $A_1^{\text{ext}} \cup A_1^{\text{ins}}$  is more likely to arise than  $A_2^{\text{ext}} \cup A_1^{\text{ins}}$  under  $Pr_2^{\text{ext}}(\cdot, \cdot)$ . Suppose further that moving from  $H_1$  to  $H_2$  makes  $A_2^{\text{ext}} \cup A_1^{\text{ins}}$  no longer preferred to  $I$  (while it is preferred under  $H_1$ ). Then external resilience falls under  $Pr_2^{\text{ext}}$  relative to  $Pr_1^{\text{ext}}$ , because the deviation that becomes newly safe receives more probability weight under  $Pr_2^{\text{ext}}$ .



Ibn Khaldun argued that nomadic tribes had a much higher level of “social cohesion” than urban civilizations, and that this strong social cohesion facilitated the conquest of urban civilizations by nomadic tribes. Our model microfound the higher social cohesion of nomadic tribes through their relative poverty compared to urban civilizations, which generates a high power-to-resources ratio. It is therefore easier for nomadic tribes to form a coalition to plunder cities, which is a repeated pattern in the pre-modern world. Similar logic can apply to communist revolutions ([Morishima \(1974\)](#); [Roemer \(1980\)](#); [Roemer \(1981\)](#); [Brewer \(2002\)](#)), where increasing inequality widens the power-to-resources gap, thereby incentivizing the proletariat to rebel against the capitalists. Importantly, our analysis may provide a clue to understanding the oligarchic tendencies of these plundering coalitions. Our model may explain why successful nomadic conquerors and communist parties, even when starting as movements of radical equality, eventually evolved into strictly hierarchical structures.

In future research, our framework may be useful as a methodological approach for studying the resilience of coalitions to exogenous changes in players’ characteristics and in the environment governing coalition formation. Moreover, although we study how coalitions respond to changes in power, resources, and plundering technology, these objects are exogenous in our model. Endogenizing them could be informative and suggests several extensions. A natural extension is to allow players to invest in power prior to coalition formation, which would clarify how the initial distribution of resources shapes power investment incentives and, ultimately, the ruling coalition. Another extension is to study environments in which the ruling coalition receives an exogenous flow of resources in addition to exploiting outsiders.

Another extension is to endogenize the plundering technology—interpreted as property-rights protection—in a dynamic version of our framework in which ruling coalitions can invest in institutions over time. This would contribute to the literature on the emergence and evolution of property-rights protection ([Andolfatto \(2002\)](#); [Hafer \(2006\)](#); [Diermeier et al. \(2017\)](#)) from the perspective of resilience. Moreover, institutions are persistent in many settings, so early institutional choices can have long-lasting effects on subsequent political and economic outcomes ([Persson \(2002\)](#); [Michalopoulos and Papaioannou \(2013\)](#); [Lowe et al. \(2017\)](#)). Finally, incorporating networks into the coalition-formation process is another promising direction. For example, [König et al. \(2017\)](#) studies how a network of military alliances affects conflict intensity. Extending our model along these lines could clarify how players’ connections shape political alliances and their resilience.

## Appendix A Proofs

*Proof of Lemma 1.* Fix  $u \in \mathbb{R}$ . Player  $i$ 's indifference curve at utility level  $u$  is the level set

$$I(u) := \{(P, X) \in \mathbb{R}^2 : G_i(P, X) = u\}.$$

To see that  $I(u)$  is closed, take any sequence  $\{(P_k, X_k)\}_{k \geq 1} \subset I(u)$  with  $(P_k, X_k) \rightarrow (P, X)$ . By continuity of  $G_i$ ,

$$G_i(P, X) = \lim_{k \rightarrow \infty} G_i(P_k, X_k) = \lim_{k \rightarrow \infty} u = u,$$

so  $(P, X) \in I(u)$ .

Next, we show that indifference curves are strictly increasing. Let  $P' > P$  and  $X' < X$ . By Assumption 1(1a),  $G_i(P', X') > G_i(P, X')$ , and by Assumption 1(1b),  $G_i(P, X') > G_i(P, X)$ . Hence  $G_i(P', X') > G_i(P, X)$ .

Suppose, toward a contradiction, that  $I(u)$  is not strictly increasing. Then there exist  $(P_1, X_1), (P_2, X_2) \in I(u)$  with  $P_2 > P_1$  and  $X_2 \leq X_1$ . If  $X_2 = X_1$ , then Assumption 1(1a) implies  $G_i(P_2, X_2) = G_i(P_2, X_1) > G_i(P_1, X_1)$ , contradicting  $G_i(P_2, X_2) = G_i(P_1, X_1) = u$ . If  $X_2 < X_1$ , then by the monotonicity implication above,  $G_i(P_2, X_2) > G_i(P_1, X_1)$ , again a contradiction. Therefore, along any indifference curve, an increase in power must be accompanied by an increase in resources, so the curve is strictly increasing.  $\square$

*Proof of Proposition 1.* Define, for each  $I \in \mathcal{W}$ ,

$$\phi(I) := \arg \max_{W \in \mathcal{W}} G(W).$$

Since  $\mathcal{W}$  is finite and non-empty,  $\arg \max_{W \in \mathcal{W}} G(W)$  is well-defined and non-empty, so  $\phi(I) \neq \emptyset$ , establishing the first part of Axiom 1. Moreover, by construction  $\phi(I) \subseteq \mathcal{W}$ , so Axiom 2 holds.

If  $I' \in \phi(I)$ , then  $I' \in \arg \max_{W \in \mathcal{W}} G(W)$ , so for any  $I'' \notin \phi(I)$  we have  $G(I'') < G(I')$ . Conversely, if  $G(I'') < G(I')$ , then  $I'' \notin \arg \max_{W \in \mathcal{W}} G(W)$ , hence  $I'' \notin \phi(I)$ . This is exactly Axiom 3. Finally, Assumption 1 implies  $G(N) = 0$ , while for any  $W \in \mathcal{W} \setminus \{N\}$  we have  $G(W) > 0$ . Hence  $N$  cannot be a maximizer of  $G$  over  $\mathcal{W}$ , so  $N \notin \phi(I)$  and the second part of Axiom 1 holds. This proves existence.

For uniqueness, suppose there is another mapping  $\phi'$  satisfying Axioms 1–3. Fix  $I \in \mathcal{W}$  and take  $I'' \in \phi'(I)$ . If  $I'' \notin \arg \max_{W \in \mathcal{W}} G(W)$ , let  $I' \in \arg \max_{W \in \mathcal{W}} G(W)$  (non-empty by the argument above). Then  $G(I'') < G(I')$ , so by Axiom 3 for  $\phi'$  we must have  $I'' \notin \phi'(I)$ , a contradiction. Hence  $\phi'(I) \subseteq \arg \max_{W \in \mathcal{W}} G(W)$ , i.e.  $\phi'(I) \subseteq \phi(I)$ .

Conversely, suppose  $I' \in \phi(I)$  but  $I' \notin \phi'(I)$ . By Axiom 1, there exists  $I'' \in \phi'(I)$ .

Since  $I'' \in \phi'(I)$  and  $I' \notin \phi'(I)$ , Axiom 3 implies  $G(I'') > G(I')$ , which contradicts  $I' \in \phi(I) = \arg \max_{W \in \mathcal{W}} G(W)$ . Therefore  $\phi(I) \subseteq \phi'(I)$ , and thus  $\phi = \phi'$ .

For the second statement, Assumption 3 implies that  $G$  takes distinct values on distinct coalitions in  $\mathcal{W}$ , so  $\arg \max_{W \in \mathcal{W}} G(W)$  is a singleton. Hence  $\phi$  is single-valued. This completes the proof.  $\square$

*Proof of Proposition 2(1).* Fix  $I_0 \in \mathcal{W}$  and  $\beta > \frac{1}{2}$ . Let  $I \in \phi(I_0) = \arg \max_{W \in \mathcal{W}} G(W)$  (Proposition 1). If  $W, W' \in \mathcal{W}$  and  $W \cap W' = \emptyset$ , then  $P_{W \cup W'} = P_W + P_{W'} \geq 2\beta P_N > P_N$ , contradicting  $P_{W \cup W'} \leq P_N$ . Hence any two winning coalitions intersect, so  $I \cap I_0 \neq \emptyset$ .

For each  $a \in I_0$ , fix  $\psi(a) \in \arg \max\{G(W) : W \in \mathcal{W}, a \in W\}$ . For any history  $h$ , let  $A^-(h)$  be the set of agenda-setters already used (including the current one at a voting node), and let  $R(h) := I_0 \setminus A^-(h)$  be the remaining agenda-setters. Write  $R_I(h) := R(h) \cap I$ . For each history  $h$  with  $R_I(h) = \emptyset$ , fix once and for all an SPE of the continuation subgame starting at  $h$  (existence follows from finiteness of the game).

Define  $\sigma^I$  as follows. If  $a \in I_0$  is selected to propose at  $h$ ,

$$\mathcal{P}^a(h) = \begin{cases} I & \text{if } a \in I, \\ \psi(a) & \text{if } a \notin I. \end{cases}$$

At any voting history  $h$  on a proposal  $\mathcal{P}$  and any voter  $v \in \mathcal{P}$ : (i) if  $\mathcal{P} = I$ , then  $v$  votes YES; (ii) if  $\mathcal{P} \neq I$  and  $R_I(h) \neq \emptyset$ , then any  $v \in I \cap \mathcal{P}$  votes NO (others arbitrary); (iii) if  $R_I(h) = \emptyset$ , players follow the fixed continuation SPE for  $h$ .

While  $R_I(h) \neq \emptyset$ , any proposal that reaches voting must be in  $\mathcal{W}$ ; if  $\mathcal{P} \in \mathcal{W}$  and  $\mathcal{P} \neq I$ , then  $\mathcal{P} \cap I \neq \emptyset$  by overlap, so some voter in  $I \cap \mathcal{P}$  is called and votes NO, implying  $\mathcal{P}$  is rejected. Since agenda-setters are drawn from  $I_0$  without replacement and  $I \cap I_0 \neq \emptyset$ , eventually some  $a \in I \cap I_0$  is selected, proposes  $I$ , and  $I$  is accepted. Thus  $\sigma^I$  yields ruling coalition  $I$ .

Fix any history  $h$  with  $R_I(h) \neq \emptyset$  and a voting node on  $\mathcal{P} \neq I$ . Pick  $i \in I \cap \mathcal{P}$  (nonempty by overlap). If  $i$  follows  $\sigma^I$  and votes NO, play continues and (under  $\sigma^I$ ) the eventual outcome is  $I$ , giving  $U_i(I)$ . If  $i$  deviates to YES, either (a) some other voter in  $I \cap \mathcal{P}$  votes NO so the proposal is still rejected and the outcome remains  $I$ , or (b)  $i$  is pivotal and  $\mathcal{P}$  is accepted, in which case Assumption 2 implies  $U_i(\mathcal{P}) = x_i + g(i)G(\mathcal{P}) \leq x_i + g(i)G(I) = U_i(I)$  since  $G(I) \geq G(\mathcal{P})$  by choice of  $I$ . Hence voting NO is (weakly) optimal at any such node.

If  $\mathcal{P} = I$ , voting YES yields  $U_i(I)$  immediately. If  $i$  deviates to NO, the game moves to some continuation outcome  $J \in \mathcal{W}$ . If  $i \in J$ , then  $U_i(J) = x_i + g(i)G(J) \leq U_i(I)$  since  $G(J) \leq G(I)$ . If  $i \notin J$ , then  $U_i(J) = x_i + w_i(J) < x_i < U_i(I)$  by Assumption 1(2) and

$g(i)G(I) > 0$ .

Now consider an agenda-setting node at  $h$  with  $R_I(h) \neq \emptyset$  and proposer  $a \in I_0$ . If  $a \in I$ , proposing  $I$  yields  $U_a(I)$ ; any deviation to  $\mathcal{P} \neq I$  that reaches voting is rejected by an insider veto as above, so it cannot improve upon proposing  $I$ . If  $a \notin I$ , any winning proposal overlaps  $I$  and is vetoed while  $R_I(h) \neq \emptyset$ , so the proposer cannot affect the continuation outcome; proposing  $\psi(a)$  is therefore a best response.

In subgames with  $R_I(h) = \emptyset$ ,  $\sigma^I$  coincides with an SPE by construction. Therefore  $\sigma^I$  is an SPE of the full game and produces  $I$ .  $\square$

*Proof of Proposition 2(2).* Assume  $\beta \in (\frac{1}{2}, 1]$  and  $\phi(I_0) = \{I\}$ . By Proposition 1 and Assumption 3,

$$G(I) > G(W) \quad \text{for all } W \in \mathcal{W} \setminus \{I\}. \quad (\text{A.1})$$

For any  $i \in I$  and any  $W \in \mathcal{W}$  with  $i \in W$ , Assumption 2 gives  $w_i(W) = g(i)G(W)$ , hence

$$U_i(I) > U_i(W) \quad \text{for all } i \in I \text{ and all } W \in \mathcal{W} \setminus \{I\} \text{ with } i \in W. \quad (\text{A.2})$$

Moreover, for any  $W \in \mathcal{W} \setminus \{N\}$  and any  $i \notin W$ , Assumption 1(2) gives  $U_i(W) = x_i + w_i(W) < x_i$ . Finally, since  $\beta > \frac{1}{2}$ , any two winning coalitions intersect (as in part (1)).

For any history  $h$ , let  $R(h) \subseteq I_0$  be the remaining agenda-setters, define  $R_I(h) := R(h) \cap I$  and  $k(h) := |R_I(h)|$ . We prove by induction on  $k(h)$  that in any subgame starting at  $h$  with  $k(h) \geq 1$ , every SPE yields ruling coalition  $I$ .

If  $k(h) = 1$ , let  $a$  be the unique agenda-setter in  $R_I(h)$ . Consider any SPE of the subgame at  $h$ . If  $a$  proposes  $I$ , then at any voting node on  $I$ , any voter  $i \in I$  weakly prefers YES: a unilateral NO rejects  $I$  and moves to a continuation with  $k = 0$ , in which the eventual ruling coalition is some  $J \in \mathcal{W}$ . If  $i \in J$  and  $J \neq I$ , then  $U_i(I) > U_i(J)$  by (A.2); if  $i \notin J$ , then  $U_i(J) < x_i < U_i(I)$ . Hence  $I$  is accepted. If instead  $a$  proposes  $W \neq I$ , then either  $W$  is accepted or rejected. If  $W$  is accepted and  $a \in W$ , then (A.2) implies  $U_a(I) > U_a(W)$ , so  $a$  profitably deviates to proposing  $I$ . If  $W$  is accepted and  $a \notin W$ , then  $U_a(W) < x_a < U_a(I)$  by Assumption 1(2) and  $g(a)G(I) > 0$ , so again deviating to propose  $I$  is profitable. If  $W$  is rejected, deviating to propose  $I$  yields acceptance and payoff  $U_a(I)$ . Thus in any SPE,  $a$  proposes  $I$  and the ruling coalition is  $I$ .

Now fix  $k \geq 2$  and assume the statement holds for all smaller values. Consider a subgame starting at  $h$  with  $k(h) = k$ , and fix any SPE. Suppose for contradiction that some  $W \in \mathcal{W}$  with  $W \neq I$  is accepted along the equilibrium path at some history  $\tilde{h}$ . By overlap,  $W \cap I \neq \emptyset$ . Let  $i \in W \cap I$  be the first member of  $W \cap I$  (under the realized voting order) who is called to vote on  $W$  at  $\tilde{h}$ . Since unanimity is required, if  $i$  votes

NO then  $W$  is rejected immediately. After this rejection the current proposer is removed from  $R(\cdot)$ , so the continuation begins at some history  $h'$  with  $k(h') \geq k - 1 \geq 1$ . Because we are in an SPE, the continuation strategies form an SPE of the subgame at  $h'$ , and by the induction hypothesis its outcome is  $I$ . Thus, by deviating to NO, player  $i$  obtains  $U_i(I)$ , whereas by not deviating she obtains  $U_i(W)$ . Since  $i \in I \cap W$  and  $W \neq I$ , (A.2) implies  $U_i(I) > U_i(W)$ , a profitable deviation, contradicting equilibrium. Therefore no  $W \neq I$  can be accepted in any SPE play from  $h$ .

It follows that in any SPE from  $h$ , the first accepted proposal must be  $I$ ; when  $I$  is proposed, the same voting argument as in the base case implies it is accepted. Hence every SPE of the subgame at  $h$  yields ruling coalition  $I$ .

Finally, since  $I_0 \in \mathcal{W}$  and  $I \in \mathcal{W}$ , overlap gives  $I_0 \cap I \neq \emptyset$ , so at the initial history  $h_0$  we have  $k(h_0) \geq 1$  and therefore every SPE of the full game yields ruling coalition  $I$ .  $\square$

*Proof of Proposition 4.* Fix a ruling coalition  $I$  and two members  $i, j \in I$  with  $p_i > p_j$  and  $x_i < x_j$ . Let the post-exchange characteristics be

$$p_{i'} = p_i - \Delta p, \quad x_{i'} = x_i + \Delta x, \quad p_{j'} = p_j + \Delta p, \quad x_{j'} = x_j - \Delta x,$$

where  $0 < \Delta p \leq (p_i - p_j)/2$  and  $0 < \Delta x \leq (x_j - x_i)/2$ . Hence  $p_{i'} \geq p_{j'}$  and  $x_{i'} \leq x_{j'}$ , and the aggregate characteristics of the ruling coalition are unchanged:

$$P_I = P_{I'} \quad \text{and} \quad X_I = X_{I'}.$$

Let  $H_I(\cdot)$  denote the indifference curve through  $(P_I, X_I)$ .

*Step 1 (best insider coalitions and what moves).* Before the exchange, no best insider sub-coalition can contain  $j$  but not  $i$ . Indeed, if  $A \subseteq I$  contains  $j$  and excludes  $i$ , then replacing  $j$  by  $i$  yields a coalition with strictly higher power and strictly lower internal resources, contradicting Definition 3. After the exchange, the same conclusion holds whenever  $(p_{i'}, x_{i'}) \neq (p_{j'}, x_{j'})$ ; on the knife-edge case  $(p_{i'}, x_{i'}) = (p_{j'}, x_{j'})$ , any best set can be chosen so that no best coalition contains  $j'$  without  $i'$  (since swapping produces the same  $(P, X)$ ).

Therefore, both before and after the exchange, every best insider sub-coalition is of one of the following types:

1. contains  $i$  but not  $j$ ;
2. contains both  $i$  and  $j$ ;
3. contains neither  $i$  nor  $j$ .

Only type-(1) coalitions move. Specifically, if  $A$  is type-(1), then after the exchange it becomes

$$(P'_A, X'_A) = (P_A - \Delta p, X_A + \Delta x),$$

while type-(2) and type-(3) coalitions do not change.

*Step 2 (weakening an insider coalition expands its external safe area).* For any insider coalition  $A \subseteq I$ , define its shifted boundary

$$H_A(P) := H_I(P + P_A) - X_A,$$

and the associated external safe area

$$\mathcal{S}_A^{\text{ext}} := \{(P, X) \in \mathbb{R}_{++}^2 : P < \beta P_N, X > H_A(P)\}.$$

(Equivalently,  $(P, X) = (P_{A^{\text{ext}}}, X_{A^{\text{ext}}}) \in \mathcal{S}_A^{\text{ext}}$  iff  $G(I) > G(A \cup A^{\text{ext}})$ .)

Suppose  $A$  is replaced by another insider coalition  $\tilde{A}$  with  $P_{\tilde{A}} \leq P_A$  and  $X_{\tilde{A}} \geq X_A$  (at least one inequality weakly strict). Since  $H_I$  is strictly increasing in  $P$  (Lemma 1), for every  $P$ ,

$$H_{\tilde{A}}(P) = H_I(P + P_{\tilde{A}}) - X_{\tilde{A}} \leq H_I(P + P_A) - X_A = H_A(P).$$

Hence  $\mathcal{S}_A^{\text{ext}} \subseteq \mathcal{S}_{\tilde{A}}^{\text{ext}}$  (with strict inclusion if at least one inequality is strict). In particular, every type-(1) best insider coalition becomes weaker and more resource-heavy after the exchange, so its associated external safe area weakly expands; type-(2) and type-(3) safe areas are unchanged.

*Step 3 (intersection, new best coalitions, and internal stability).* External resilience is the intersection of the external safe areas across best insider coalitions:

$$\mathcal{S}_I^{\text{ext}} = \bigcap_{A \in \mathcal{A}_I} \mathcal{S}_A^{\text{ext}}, \quad \mathcal{S}_{I'}^{\text{ext}} = \bigcap_{A \in \mathcal{A}_{I'}} \mathcal{S}_A^{\text{ext}}.$$

Consider the change from  $\mathcal{A}_I$  to  $\mathcal{A}_{I'}$ .

**(a) Coalitions that remain best.** For every  $A \in \mathcal{A}_I \cap \mathcal{A}_{I'}$ , Step 2 implies  $\mathcal{S}_A^{\text{ext}}$  weakly expands if  $A$  is type-(1) and is unchanged otherwise.

**(b) Coalitions that cease to be best.** If some  $A \in \mathcal{A}_I$  is no longer best after the exchange, then its safe area is removed from the intersection, which can only weakly enlarge the intersection.

**(c) Newly best coalitions.** Let  $C \in \mathcal{A}_{I'} \setminus \mathcal{A}_I$  be a newly best insider coalition. We claim  $C$  cannot be type-(1). Indeed, every type-(1) coalition shifts by the same vector

$(-\Delta p, +\Delta x)$ , so dominance relations among type-(1) coalitions are preserved; moreover, relative to any coalition that does not move, a type-(1) coalition becomes weakly *less* powerful and weakly *more* resource-heavy. Hence a type-(1) coalition cannot become newly undominated. Therefore  $C$  is type-(2) or type-(3), so it does not move.

Since  $C$  was not best before the exchange, there exists at least one *pre-exchange* best coalition  $A \in \mathcal{A}_I$  that strictly dominates it:

$$P_A > P_C, \quad X_A < X_C.$$

Necessarily such an  $A$  must be type-(1) (otherwise the dominance would persist after the exchange and  $C$  could not become best). By Step 2 applied to  $(A, C)$  we have  $\mathcal{S}_A^{\text{ext}} \subseteq \mathcal{S}_C^{\text{ext}}$ . Since  $\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_A^{\text{ext}}$ , it follows that  $\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_C^{\text{ext}}$ . Thus adding  $C$  to the intersection cannot exclude any point that was externally safe before.

Putting (a)–(c) together, we obtain

$$\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_{I'}^{\text{ext}},$$

so external resilience weakly increases.

**Internal stability.** Because  $(P_I, X_I)$  is unchanged, the internal safe region  $\{(P, X) : X > H_I(P)\}$  is unchanged. Any insider coalition that moves does so left/up, making  $X > H_I(P)$  easier to satisfy; coalitions that do not move are unaffected. Hence no new profitable internal secession is created, so internal stability is preserved.

Therefore, the exchange weakly increases the external resilience of the ruling coalition, proving the proposition.  $\square$

*Proof of Proposition 5.* Fix a ruling coalition  $I$  and let  $H_I(\cdot)$  be the indifference curve through  $(P_I, X_I)$ , so  $X_I = H_I(P_I)$ . Take any non-trivial best insider sub-coalition  $A^{\text{ins}} \in \mathcal{A}_I \setminus \{I\}$  and write  $(P_A, X_A) := (P_{A^{\text{ins}}}, X_{A^{\text{ins}}})$ .

Since  $I$  is a ruling coalition, internal stability implies  $G(I) > G(A^{\text{ins}})$ , equivalently

$$X_A > H_I(P_A). \tag{A.3}$$

For any  $K \subseteq I$ , define the translated boundary and external safe region

$$H_I^K(P) := H_I(P + P_K) - X_K, \quad \mathcal{S}_K^{\text{ext}} := \{(P, X) \in \mathbb{R}_{++}^2 : P < \beta P_N, X > H_I^K(P)\}.$$

In particular,  $H_I^I(P) = H_I(P + P_I) - X_I$  and  $H_I^A(P) = H_I(P + P_A) - X_A$ .

We claim that for every  $P \geq 0$ ,

$$H_I^I(P) > H_I^A(P), \quad (\text{A.4})$$

which implies  $\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_{A^{\text{ins}}}^{\text{ext}}$ . To prove (A.4), note that  $P_I > P_A$  and set  $\Delta := P_I - P_A > 0$ . Then

$$H_I^I(P) - H_I^A(P) = [H_I(P + P_A + \Delta) - H_I(P + P_A)] - (X_I - X_A).$$

Because  $H_I$  is strictly convex, the increment  $H_I(t + \Delta) - H_I(t)$  is weakly increasing in  $t$ . With  $t = P + P_A \geq P_A$ ,

$$H_I(P + P_A + \Delta) - H_I(P + P_A) \geq H_I(P_A + \Delta) - H_I(P_A) = H_I(P_I) - H_I(P_A).$$

Using  $X_I = H_I(P_I)$  and (A.3), we have  $H_I(P_I) - H_I(P_A) > X_I - X_A$ , hence  $H_I^I(P) - H_I^A(P) > 0$  for all  $P \geq 0$ , proving (A.4).

Therefore, for every  $A^{\text{ins}} \in \mathcal{A}_I \setminus \{I\}$ ,  $\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_{A^{\text{ins}}}^{\text{ext}}$ . Since  $I \in \mathcal{A}_I$ ,

$$\mathcal{S}^{\text{ext}} = \bigcap_{A^{\text{ins}} \in \mathcal{A}_I} \mathcal{S}_{A^{\text{ins}}}^{\text{ext}} = \mathcal{S}_I^{\text{ext}}.$$

Finally, any internal exchange that preserves  $(P_I, X_I)$  leaves  $H_I$  and thus  $\mathcal{S}_I^{\text{ext}}$  unchanged, so external resilience is invariant to such internal reallocations.  $\square$

*Proof of Proposition 6.* Fix a ruling coalition  $I$  and consider the finite sequence of within- $I$  exchanges in Figure 5 that keeps  $(P_I, X_I)$  fixed and terminates at a hierarchical allocation  $I' := I^T$ . Let  $I^t$  denote the allocation after  $t$  exchanges. By Proposition 4, each exchange weakly enlarges the external safe area, so

$$\mathcal{S}_I^{\text{ext}} \subseteq \mathcal{S}_{I'}^{\text{ext}}.$$

If the inclusion is strict at some intermediate step, we are done. Hence suppose  $\mathcal{S}_{I^t}^{\text{ext}} = \mathcal{S}_I^{\text{ext}}$  for all  $t < T$ .

Immediately before the last exchange, there is (by construction) a remaining mis-ordered pair  $i, j \in I$  with  $p_i^{T-1} > p_j^{T-1}$  and  $x_i^{T-1} < x_j^{T-1}$ . Consider any best insider sub-coalition  $C \in \mathcal{A}_{I^{T-1}}$ . As in the proof of Proposition 4, no best insider sub-coalition can contain  $j$  but exclude  $i$ : if  $j \in C$  and  $i \notin C$ , then replacing  $j$  by  $i$  strictly increases power and strictly decreases internal resources, contradicting the definition of  $\mathcal{A}_{I^{T-1}}$ . Therefore, the only best insider sub-coalitions whose  $(P, X)$ -location changes in the last exchange are those that contain  $i$  and exclude  $j$ .

Fix any  $A \in \mathcal{A}_{I^{T-1}}$  with  $i \in A$  and  $j \notin A$ , and let  $A'$  be its post-exchange counterpart.

By the definition of the exchange,  $p_i$  decreases and  $x_i$  increases, while  $j \notin A$ , so

$$P_{A'} < P_A \quad \text{and} \quad X_{A'} > X_A.$$

Recall the translated boundary  $H_C(P) := H_I(P + P_C) - X_C$  and the associated external safe region  $\mathcal{S}_C^{\text{ext}} := \{(P, X) \in \mathbb{R}_{++}^2 : P < \beta P_N, X > H_C(P)\}$ . Since  $H_I$  is strictly increasing, for every  $P < \beta P_N$  we have

$$H_{A'}(P) - H_A(P) = [H_I(P + P_{A'}) - H_I(P + P_A)] - (X_{A'} - X_A) < 0,$$

hence  $H_{A'}(P) < H_A(P)$  for all  $P < \beta P_N$ , and therefore

$$\mathcal{S}_A^{\text{ext}} \subsetneq \mathcal{S}_{A'}^{\text{ext}}. \tag{A.5}$$

The external safe area of the ruling coalition is  $\mathcal{S}_{I^{T-1}}^{\text{ext}} = \bigcap_{C \in \mathcal{A}_{I^{T-1}}} \mathcal{S}_C^{\text{ext}}$ . In the last exchange, (i) every best sub-coalition that does not contain  $i$  is unchanged, hence its  $\mathcal{S}_C^{\text{ext}}$  is unchanged; (ii) every best sub-coalition that contains  $i$  and excludes  $j$  expands strictly as in (A.5); (iii) any change in the best-subcoalition set can only remove dominated constraints (which weakly enlarges the intersection), and cannot create a new best sub-coalition that contains  $j$  but excludes  $i$  (by the dominance argument above). Consequently, the last exchange yields a strict expansion of the intersection whenever at least one affected boundary  $H_A$  is binding somewhere in the upper envelope  $\max_{C \in \mathcal{A}_{I^{T-1}}} H_C(P)$  on  $(0, \beta P_N)$ .

It remains to show that for sufficiently concave CES preferences (i.e.  $\rho$  sufficiently negative), such binding occurs. As  $\rho \rightarrow -\infty$ ,  $G_\rho$  converges pointwise to the Leontief aggregator  $G_{-\infty}$ , and the indifference curve through  $I$  becomes kinked; each translated boundary  $H_C(\cdot)$  inherits a single kink. Since  $\mathcal{A}_{I^{T-1}}$  is finite, the upper envelope of these kinked boundaries is piecewise and is attained by a single boundary except at finitely many kink/intersection points. In particular, some affected best sub-coalition  $A \in \mathcal{A}_{I^{T-1}}$  with  $i \in A$  and  $j \notin A$  attains the envelope at some  $P^* \in (0, \beta P_N)$  in the Leontief limit (cf. Figure 9). By continuity of  $H_I$  and hence  $H_C(\cdot)$  in  $\rho$  on compact  $P$ -sets, there exists  $\bar{\rho} < 1$  such that for all  $\rho \leq \bar{\rho}$ , the same coalition  $A$  remains binding at some  $P^* \in (0, \beta P_N)$ . For such  $\rho$ , the last exchange strictly lowers the envelope at  $P^*$  (by (A.5)), and thus strictly enlarges  $\mathcal{S}_{I^{T-1}}^{\text{ext}}$ , implying

$$\mathcal{S}_I^{\text{ext}} \subsetneq \mathcal{S}_{I'}^{\text{ext}}.$$

□

## Appendix B Examples

The following example illustrates a typical function  $G_i(\cdot)$  satisfying Assumptions 1-3 and clarifies the distinction between environments that generate inclusive versus exclusive ruling coalitions.

**Example 3.** For any  $i \in I \in \mathcal{W} \setminus \{N\}$ , consider

$$w_i(I) = G_i(I) := \left(\frac{p_i}{P_I}\right) \left(\frac{P_I}{P_N}\right)^{\alpha+1} \left(\frac{X_N}{X_I}\right), \quad (\text{B.1})$$

where  $\alpha > 0$ . The term  $\frac{p_i}{P_I}$  is  $i$ 's share of plundered resources, proportional to her relative power in the ruling coalition, and the plunder function  $\left(\frac{P_I}{P_N}\right)^{\alpha+1} \left(\frac{X_N}{X_I}\right)$  ranks ruling coalitions by the resources they extract. One can verify that  $\alpha > 0$  is required for parts (i)-(ii) of Assumption 1; if  $\alpha < 0$ , these are violated.

Normalize  $P_N = X_N = 1$ . Then

$$G_i(I) = p_i P_I^\alpha \frac{1}{X_I}.$$

Fix a payoff level  $\bar{G}_i$  and let  $I$  be a ruling coalition containing  $i$ . The indifference curve of  $i$  through  $I$  is the locus of  $(P, X)$  with  $G_i(P, X) = \bar{G}_i$ :

$$X = C_i(I) P^\alpha, \quad (\text{B.2})$$

where  $C_i(I) := \frac{p_i}{\bar{G}_i}$  and  $P \in [\beta, 1]$ . Along such an indifference curve the marginal rate of substitution between power and resources is

$$MRS_{PX} = -\alpha \frac{X}{P}.$$

The parameter  $\alpha$  governs the relative valuation of power versus resources. A higher  $\alpha$  makes indifference curves steeper: for given  $(P, X)$ , the marginal value of power relative to internal resources is higher, so players are more willing to sacrifice resources to gain power. Since  $P \in [\beta, 1]$ , a higher  $\alpha$  reduces  $P_I^\alpha$  (for  $P_I < 1$ ) and thus dampens the increase in plunder from further increases in power. This corresponds to a relatively weak plundering technology: very powerful coalitions obtain relatively modest incremental gains from additional power, and insiders are more willing to form large, inclusive ruling coalitions.

Conversely, lower values of  $\alpha$  flatten indifference curves and raise  $P_I^\alpha$  for  $P_I < 1$ , making payoffs more sensitive to power and less constrained by internal resources. This corresponds to a relatively intensive plundering technology: small, powerful coalitions can

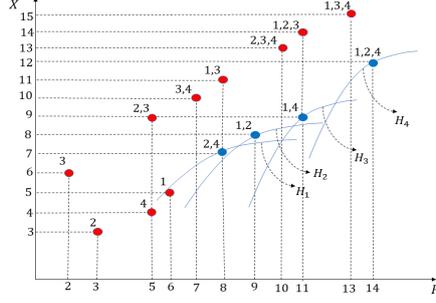


Figure 16: Ruling coalition under different indifference curves.

*extract much more from outsiders, and insiders are more reluctant to dilute power, so ruling coalitions tend to be more exclusive.*

The next example shows that, without further restrictions on the joint distribution of power and resources and the plundering function, there is no general characterization of the ruling coalition's composition. This follows, first, from Proposition 2(1), which implies that any coalition in the set of potential ruling coalitions may be the ruling coalition for some range of indifference curves; and second, from the fact that the set of potential ruling coalitions itself cannot be sharply characterized without additional structure on  $(p_i, x_i)_{i \in N}$ . In particular, there is no guarantee that the ruling coalition contains the most powerful player, the player with the fewest resources, or the player with the highest power-to-resource ratio (Figure 16).<sup>15</sup> Example 5 illustrates this point.

**Example 4.** Suppose  $N = \{1, 2, 3, 4\}$ , with

$$p_1 = 6, x_1 = 5, \quad p_2 = 3, x_2 = 3, \quad p_3 = 2, x_3 = 6, \quad p_4 = 5, x_4 = 4,$$

and let  $\beta = \frac{1}{2} + \epsilon$ . Then the set of winning coalitions is

$$\mathcal{W} = \{\{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\}.$$

As shown in Figure 16, different indifference curves select different ruling coalitions: when the indifference curve is  $H_1$ , the ruling coalition is  $\{2, 4\}$ , which excludes the player with the highest power (player 1); when it is  $H_3$ , the ruling coalition is  $\{1, 4\}$ , which excludes the player with the lowest resources; and when it is  $H_2$ , the ruling coalition excludes the player with the highest power-to-resource ratio (player 4). Thus, absent additional structure, the ruling coalition need not contain the most powerful, or the poorest player.

<sup>15</sup>Example 4 in the appendix shows that a sharper characterization is possible when powers, resources, or both are equally distributed in society.

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