Uniform and Participation-expanding Reforms in Decentralized Redistribution: Who Gains and Who Loses?*

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April 8, 2026

Abstract

Across many settings, from social transfers to education and health services, reforms that aim to make access uniformly easier for everyone often exacerbate inequality. When access to the redistributive surplus is determined through competition, such reforms increase players' efficiency in the contest but also raise the endogenous efficiency threshold for participation. I formalize this threshold-shifting mechanism in a model of decentralized redistribution, where heterogeneous efficiencies determine both who competes and how the redistributive surplus is allocated. The model delivers two main results. First, a uniform efficiency gain—i.e., equal reductions in access costs—does not expand participation. Instead, it amplifies inequality among participants by reinforcing the relative efficiency of those who were already highly efficient prior to the reform. Yet, such a reform also reduces contest-induced welfare loss, thereby giving rise to an equity–efficiency trade-off. Second, a participation-expanding reform that brings excluded players into the contest can reduce both inequality and welfare loss, particularly when: (i) the new entrants are sufficiently efficient relative to the average participant; and (ii) the Herfindahl–Hirschman Index (HHI) of endogenous contest strengths—measuring the extent to which players' efficiencies exceed the participation threshold—is sufficiently high before the reform; for example, when a few strong participants dominate the competition for redistributive surplus.

Keywords: Redistribution, Inequality, Reforms, Inefficiency
JEL Codes: D72, D74, D31, C72

*All errors and omissions remain my responsibility.

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1 Introduction

Governments often seek to broaden access to public resources through *uniform reforms*: policies that reduce access costs equally for everyone, such as digitized applications, simplified procedures, or transparency portals. Yet such reforms do not always equalize outcomes. In many settings, they have instead been followed by greater inequality in effective access. For example, biometric payment reforms in India improved payment efficiency while also generating exclusion errors for some beneficiaries (Muralidharan et al., 2016); broader financial-management reforms produced uneven gains across regions and social groups (Banerjee et al., 2020); and transparency campaigns in Uganda reduced corruption but yielded larger gains in areas with better media access (Reinikka and Svensson, 2004). Why can policies intended to equalize access instead reinforce disparities?

This paper studies one mechanism through which that can occur. I consider a model of contest-based redistribution with endogenous participation, in which agents differ in the efficiency with which they convert resources into effort. Participation depends on relative efficiency: only agents that are sufficiently strong relative to their rivals exert positive effort. In such an environment, a uniform improvement in efficiency need not broaden access. Instead, it can tighten the endogenous participation cutoff and reallocate redistribution toward agents who were already strong participants.

The paper makes one central point: the distributional effect of reform depends not only on whether frictions fall, but also on how reform changes the endogenous participation threshold. This threshold-shifting channel generates three main results. First, a uniform efficiency improvement does not expand participation. Because the participation cutoff moves with rivals' efficiency, a common reduction in inefficiency can raise the effective standard for participation and leave inactive agents excluded. Second, among active agents, the same reform reallocates redistribution toward initially stronger participants and therefore increases inequality within the active set. Third, uniform reform reduces aggregate dissipation by increasing the concentration of contest strength. Uniform reform thus generates an endogenous equity–efficiency trade-off.

The mechanism is simple. In equilibrium, participation is determined by an endogenous cutoff that depends on the composition of the active set. A uniform reform improves every agent's efficiency, but it also intensifies competition among incumbents. As a result, the participation cutoff tightens. Since this cutoff moves with rivals' efficiency, it can move by more than the direct gain enjoyed by any one inactive agent. Uniform reform then fails to broaden access even though everyone becomes individually more efficient. Among incumbents, the same change compresses weaker participants more than stronger ones in relative terms, shifting redistribution toward the top of the active set.

By contrast, reforms that relax the participation margin operate differently. When previously excluded agents become efficient enough to enter, they obtain a positive share of the contested surplus and reduce incumbents' shares. Under additional conditions, entry can also lower aggregate dissipation, particularly when entrants become sufficiently efficient or when pre-reform contest strength is concentrated among a small number of dominant participants. The paper therefore distinguishes between two policy margins that are often conflated: reducing frictions uniformly and expanding participation directly.

The paper contributes to the literature on contests with heterogeneous agents and endogenous participation, but its emphasis is different from existing work. Prior models show how heterogeneity shapes equilibrium entry, effort, and dissipation. My focus is on how policy changes the participation cutoff itself. The object of interest is therefore not merely how equilibrium outcomes respond to a change in one agent's cost or ability, but how reforms that shift frictions more broadly alter access, redistribution, and dissipation through the endogenous participation margin. The paper does not claim that all unequal effects of administrative reform operate through this channel. Rather, it isolates one mechanism that is likely to matter in settings where access depends on costly initiative, information, or bureaucratic navigation.

The rest of the paper proceeds as follows. Section 1.1 reviews the related literature and presents motivating evidence. Section 2 introduces the model. Section 2.1 characterizes equilibrium participation, effort, redistribution, and aggregate dissipation. Section 2.2 analyzes the effects of uniform and participation-expanding reforms. Section 3 considers robustness and extensions. Section 4 concludes.

1.1 Related Literature

This paper contributes to three related strands of research in public economics, development, and political economy. Its central contribution is to show how uniform reforms can amplify inequality through an endogenous threshold-shifting mechanism: by changing the participation cutoff in contest-based redistribution, such reforms can intensify exclusion and reallocate surplus toward already strong participants.

First, the paper relates to the literature on contests and rent seeking, in which agents expend costly effort to capture a surplus (e.g., Tullock (1980); Skaperdas (1996); Konrad (2009)). Classical models typically assume fixed participation, while later work allows for heterogeneity in costs or abilities and studies how this heterogeneity affects entry, effort, and dissipation (e.g., Moldovanu and Sela (2006); Esteban and Ray (2011); Gradstein (1995); Nti (1999)). This paper builds on that literature by making the participation threshold itself policy-relevant. Unlike models in which the participation cutoff is characterized for a given distribution of costs or abilities, the object of interest here is how policy shifts that cutoff by changing frictions across agents simultaneously. The comparative statics therefore concern reform-induced movement in the participation margin itself, rather than equilibrium entry for a fixed environment. The paper therefore studies how policy changes equilibrium access, redistribution, and dissipation through the participation margin.

Second, the paper connects to work on unequal access to public resources in low-capacity or unequal institutional environments. A large empirical literature shows that administrative and technological reforms intended to reduce average frictions often disproportionately benefit already advantaged groups. In Uganda, transparency campaigns improved school funding outcomes mainly where local actors were better positioned to act on information (Reinikka and Svensson (2004); Bardhan and Mookherjee (2006)). In India, digitization and payment reforms improved delivery on average while excluding or disadvantaging groups with weaker access to infrastructure, information, or administrative capacity (Dutta et al. (2014); Olken (2007); Olken and Pande (2012)). More recent evidence points to similar patterns in broader settings, including financial inclusion and service access (Fonseca and Matray (2024); Tu et al. (2025)). This paper provides a simple theoretical mechanism for these patterns. Even when reforms reduce frictions symmetrically, they may raise the effective bar for participation and thereby widen distributive gaps.

The paper also highlights a contrasting implication: reforms that expand participation can reduce both inequality and dissipation. This result connects to evidence on inclusion-oriented interventions, such as local monitoring, targeted mobilization, and participation subsidies, which have been shown to broaden effective access and compress disparities (Bardhan and Mookherjee (2000); Björkman and Svensson (2009); Casey et al. (2012); Banerjee et al. (2015); Banerjee et al. (2019); Andrews et al. (2021)). The broader implication is that efficiency-enhancing reforms need not be equalizing unless they also relax barriers at the participation margin.

Finally, the paper relates to work on the relationship between wealth, effective capacity, and inequality. In many environments, wealth and contest capacity are positively correlated, so that already advantaged agents are also better able to convert resources into influence. This logic is related to models in which wealth shapes investment opportunities, productivity, or political influence (Aghion and Bolton (1997); Galor and Moav (2004); Aghion et al. (2015)), as well as to recent work showing that market power or heterogeneity in returns can magnify inequality (Aghion et al. (2023); Daminato and Pistaferri (2024); Impullitti and Rendahl (2025)). A related idea appears in Esteban and Ray (1999), where stronger alignment between wealth and ability intensifies conflict and inequality. In that broader perspective, the mechanism studied here helps clarify why identical efficiency-enhancing reforms can have different distributive effects depending on who becomes relatively stronger when frictions fall.

2 Model

This section introduces the benchmark contest model. The environment is intentionally simple: it isolates how heterogeneity in effort costs shapes equilibrium participation, redistribution, and aggregate dissipation. Section 3 shows that the main qualitative mechanism extends beyond the lottery benchmark used here.

Consider a finite set of players $\mathcal{N} = \{1, \ldots, N\}$. Each player i has initial wealth $w_i > 0$ and simultaneously chooses non-negative effort $e_i \geq 0$. Effort determines each player's share of a common redistributive surplus. Players differ in the marginal cost of effort and therefore face a trade-off between expending costly effort to capture a larger share of the surplus and remaining passive to avoid dissipation.

In the benchmark specification, player i's share of the contested surplus is e_i/E_N, where $E_N := \sum_j e_j$ denotes total effort. This is the standard lottery contest success function: each unit of effort acts like a lottery ticket, so the probability of capturing the surplus is proportional to relative effort (Tullock, 1980; Skaperdas, 1996; Konrad, 2009). When $E_N = 0$, I adopt the convention that $e_i/E_N := 1/N$ and $(\sum_{j \neq i} e_j)/E_N := (N-1)/N$.

To isolate the role of cost heterogeneity, I assume a linear cost structure. Effort has two components. First, a fraction $\rho \in [0, 1]$ is deducted directly from wealth, capturing institutional frictions such as delays, bribes, or processing costs. Residual wealth is therefore $R_i := w_i - \rho e_i$. Second, player i incurs a private marginal effort cost $\kappa_i e_i$, which may reflect time costs, opportunity costs, or risk. The total marginal cost of effort is thus $\kappa_i + \rho$. Distinguishing between the common component ρ and the idiosyncratic component κ_i is not essential for the formal results, but it is useful for interpretation.

Redistribution is decentralized and determined by strategic effort. Under a uniform rule, the contested pool has two components: an endogenous component equal to a fraction $\phi \in (0, 1]$ of aggregate residual wealth, $S_R := \sum_j R_j$, and an exogenous prize V. Player i's expected utility is

$$U_i(e_i, e_{-i}) = \frac{e_i}{E_N}\left[R_i + \phi \sum_{j \neq i} R_j + V\right] + \frac{\sum_{j \neq i} e_j}{E_N}(1-\phi)R_i - \kappa_i e_i. \tag{2.1}$$

The own-residual term inside the win state and the residual wealth retained in the lose state combine to yield the secured component $(1-\phi)R_i$, so the payoff can be written more compactly as

$$U_i(e_i, e_{-i}) = (1-\phi)R_i + \frac{e_i}{E_N}(\phi S_R + V) - \kappa_i e_i, \tag{2.2}$$

where $S_R = \sum_{j \in \mathcal{N}} R_j$. Thus each player keeps the non-contested part of her residual wealth, competes for the contested pool $\phi S_R + V$, and pays a private effort cost.

Substituting $R_i = w_i - \rho e_i$ into (2.2) shows that the model is strategically equivalent to a contest with a fixed prize. Since $S_R = S_W - \rho \sum_i e_i$, where $S_W := \sum_i w_i$, the contested pool is $\phi S_R + V = \phi(S_W - \rho \sum_i e_i) + V$. The $-\rho e_i$ terms then cancel, so the effective prize is constant and equal to $P := \phi S_W + V$. The benchmark model can therefore be written as a contest over a fixed surplus while retaining the interpretation that effort both helps capture redistribution and dissipates resources.

Finally, I abstract from liquidity constraints by assuming that effort costs do not depend directly on initial wealth. In particular, players' marginal effort costs are independent of w_i.[1] This assumption lets the analysis focus on how policy-driven changes in contest efficiency affect participation, redistribution, and aggregate dissipation.

2.1 Equilibrium Characterization

This section characterizes the static equilibrium and the endogenous participation margin. I begin by defining players' inefficiency.

Definition 1 (Inefficiency)**.** *Player i's inefficiency is $a_i := \kappa_i + \rho$, where κ_i is the private marginal cost of effort and ρ is the common institutional friction.*

The parameter a_i captures the total marginal cost of converting effort into influence. Lower values of a_i therefore correspond to greater contest efficiency. In equilibrium, only players with sufficiently low inefficiency exert positive effort.

Definition 2 (Active Set)**.** *The* active set $\mathcal{L} \subseteq \mathcal{N}$ *consists of players who exert positive effort in equilibrium. A player $i \in \mathcal{N}$ is active if and only if $a_i < \tilde{a}$, where the endogenous participation threshold is*

$$\tilde{a} := \frac{1}{L-1} \sum_{j \in \mathcal{L}} a_j, \qquad L := |\mathcal{L}|.$$

Equivalently, $\mathcal{L} = \{i \in \mathcal{N} : a_i < \tilde{a}\}$.

The threshold $\tilde{a}$ is self-referential, since it depends on the active set that it itself determines. The next lemma shows that this fixed-point problem has a unique solution.

Lemma 1 (Existence and Uniqueness of the Active Set and Threshold)**.** *Let $\mathcal{N}$ be a finite population of $N \geq 2$ players with inefficiencies $\{a_i\}_{i \in \mathcal{N}}$. Then there exists a unique threshold $\tilde{a} \in \mathbb{R}$ and a unique non-empty active set $\mathcal{L} \subseteq \mathcal{N}$ with $L \geq 2$ such that $\tilde{a} = \frac{1}{L-1} \sum_{j \in \mathcal{L}} a_j$ and $\mathcal{L} = \{i \in \mathcal{N} : a_i < \tilde{a}\}$.*

Given the active set, define each player's contest strength as follows.

[1] This excludes wealth from the *cost* side of the model, but not from the *prize* side. Equilibrium effort still scales with aggregate wealth through $P = \phi S_W + V$, and hence indirectly with each player's wealth via S_W.

Definition 3 (Contest Strength). *For each player $i \in \mathcal{N}$, the* contest strength *is $c_i := \max\{\tilde{a} - a_i, 0\}$, where $\tilde{a}$ is the participation cutoff. Total contest strength is $\mathcal{C} := \sum_{j \in \mathcal{L}} c_j$.*[2]

Proposition 1 (Equilibrium Participation, Effort, and Utility). *There exists a unique pure-strategy Nash equilibrium with the following properties:*

(i) Participation. Player $i \in \mathcal{N}$ exerts positive effort if and only if $i \in \mathcal{L}$, where $\mathcal{L}$ and $\tilde{a}$ are uniquely defined as in Definition 2.

(ii) Effort. Equilibrium effort is

$$e_i = \begin{cases} \left(\frac{\tilde{a} - a_i}{\tilde{a}^2}\right)(\phi S_W + V) & \text{if } i \in \mathcal{L}, \\ 0 & \text{if } i \notin \mathcal{L}. \end{cases} \tag{2.3}$$

(iii) Utility. Equilibrium utility is

$$U_i = (1 - \phi) w_i + s_i(\phi S_W + V), \tag{2.4}$$

where the net redistribution share is

$$s_i := \left(\frac{c_i}{\mathcal{C}}\right)^2,$$

with c_i and $\mathcal{C}$ as in Definition 3.[3]

The proof is given in Appendix 5. Proposition 1 implies that equilibrium participation is selective and determined entirely by relative inefficiency. In particular, a player enters if and only if her inefficiency lies below a scaled average of active players' inefficiencies. When $L > 2$, the condition $a_i < \frac{1}{L-1} \sum_{j \in \mathcal{L}} a_j$ is equivalent to $a_i < \frac{1}{L-2} \sum_{j \in \mathcal{L} \setminus \{i\}} a_j$. Access to redistribution is therefore endogenous and comparative: only sufficiently efficient players participate.

The proposition also implies that no equilibrium can have fewer than two active players. If only one player exerted positive effort, that player could profitably reduce effort while retaining the entire contested surplus. If all players exerted zero effort, any player could profitably deviate by exerting an arbitrarily small amount of effort and capturing the surplus.

Part (ii) shows that equilibrium effort is linear in contest strength $c_i = \tilde{a} - a_i$ for active players and zero for inactive players. Part (iii) shows that post-contest utility consists

[2] In equilibrium, $\mathcal{C} = \sum_{j \in \mathcal{L}} (\tilde{a} - a_j) = \tilde{a}$, as shown in the proof of Proposition 1.

[3] Here

$$s_i = \left(\frac{c_i}{\mathcal{C}}\right)^2$$

denotes the net share of the contestable surplus accruing to player i in equilibrium, after endogenous effort costs, whereas $c_i/\mathcal{C}$ is the gross probability share before costs.

of secured wealth, $(1-\phi)w_i$, plus a net redistribution component proportional to the player's equilibrium share.

Remark 1 (*Difference in redistribution shares among active players*). For any two active players $i, j \in \mathcal{L}$, the difference in redistribution shares is

$$\Delta s^{ij} := s_i - s_j = \frac{2(a_j - a_i)\left(\tilde{a} - \frac{a_i+a_j}{2}\right)}{\mathcal{C}^2}. \tag{2.5}$$

The term $\tilde{a} - \frac{a_i+a_j}{2}$ acts as a convexity multiplier. When a_i and a_j are both far below $\tilde{a}$, even small inefficiency differences generate disproportionately large differences in redistribution shares. Moreover, $s_i > s_j$ if and only if $a_i < a_j$, so the equilibrium ranking of redistribution shares coincides with the ranking of efficiencies. Finally, lower total contest strength $\mathcal{C}$ amplifies these differences.

Another equilibrium object of interest is aggregate dissipation, that is, the portion of the total surplus lost through costly effort.

Corollary 1 (Equilibrium Aggregate Dissipation). *In equilibrium, aggregate dissipation is*

$$\mathcal{D} = \delta(\phi S_W + V), \qquad \textit{where} \qquad \delta := 1 - \frac{\sum_{i\in\mathcal{L}} c_i^2}{\left(\sum_{i\in\mathcal{L}} c_i\right)^2} = 1 - \mathrm{HHI}(c) \in [0,1]. \tag{2.6}$$

Thus, equilibrium dissipation depends on the distribution of contest strength within the active set. Greater concentration of contest strength raises $\mathrm{HHI}(c)$ and lowers dissipation, because dominant players can sustain their position with less aggregate effort. By contrast, when contest strength is more evenly distributed, rivalry intensifies and aggregate dissipation rises.

The final implication of the benchmark equilibrium concerns who benefits from redistribution.

Remark 2 (*Winners and losers of redistribution*). In equilibrium, inactive players $i \notin \mathcal{L}$ always lose from redistribution, since they forfeit a fraction ϕ of their wealth without receiving any return: $U_i^* = (1-\phi)w_i < w_i$. An active player $i \in \mathcal{L}$ is worse off whenever her redistribution share falls short of her contribution to the contested surplus, that is, whenever

$$s_i = \left(\frac{c_i}{\mathcal{C}}\right)^2 < \frac{\phi w_i}{\phi S_W + V}. \tag{2.7}$$

However, for any given distribution of contest strengths, a sufficiently large exogenous prize V implies that every active player gains from redistribution.

2.2 Non-Monotonic Effects of Efficiency

I now study how efficiency gains affect redistribution shares and aggregate dissipation. The key objects are the equilibrium shares s_i and the dissipation factor δ. The central point is that the effects of efficiency gains are generally non-monotonic: they depend not only on who becomes more efficient, but also on how the induced change in efficiency shifts the participation cutoff and reshapes relative contest strength.

This non-monotonicity does not rely on the convexity of redistribution shares in relative strength. Rather, it is driven by two forces that arise broadly in contests with endogenous participation. First, policy may shift the participation cutoff and thereby induce entry or exit. Second, any change in the cutoff alters participants' relative contest strengths. Convexity, when present, amplifies the magnitude of these effects but does not determine their direction. Section 3 shows that this mechanism extends beyond the benchmark lottery contest.

2.2.1 Redistribution Shares

I consider three types of efficiency gains: targeted gains that leave the active set unchanged, gains that induce entry by previously inactive players, and uniform gains that apply to all players. Propositions 2–4 characterize how these reforms affect equilibrium redistribution shares. Table 1 summarizes the results.

Table 1: Direction of redistribution-share changes under different efficiency gains

Efficiency Gain Type	**Inactive**	**Weak Active**	**Strong Active**
(1) Targeted gain to inactives (no entry)	–	–	–
(2) Targeted gain to weak actives	–	↑	↓
(3) Entry of an inactive player	↑	↓	↓
(4) Uniform gain	–	↓	↑

The table highlights the central contrast. Participation-expanding reforms reduce the gap between active and inactive players by giving entrants a positive share and lowering incumbents' shares. Uniform reforms do not broaden access. Instead, they reallocate redistribution toward the strongest incumbents and increase inequality within the active set.

I begin with the comparative statics of redistribution shares with respect to individual inefficiency, holding the active set fixed. Specifically, assume there exists $\varepsilon > 0$ such that $|\tilde{a} - a_i| > \varepsilon$ for all $i \in \mathcal{N}$.

Proposition 2 (Comparative Statics of Redistribution Shares with Respect to Inefficiency)**.** *In the equilibrium characterized by Proposition 1:*

(i) Inactive players are irrelevant off margin. For any inactive player $j \notin \mathcal{L}$, marginal changes in inefficiency do not affect equilibrium redistribution shares: $\frac{\partial s_i}{\partial a_j} = 0$ for all $i \in \mathcal{N}$.

(ii) *Own efficiency matters. For any active player $i \in \mathcal{L}$, an increase in own inefficiency reduces her redistribution share:* $\frac{\partial s_i}{\partial a_i} < 0$.

(iii) *Relative strength governs redistribution. For any distinct active players $i, j \in \mathcal{L}$, an increase in a_j raises i's redistribution share:* $\frac{\partial s_i}{\partial a_j} > 0$.

The proof is in Appendix 5. Part (i) shows that off-margin inactive players are irrelevant: as long as they remain inactive, their inefficiency does not affect the participation cutoff or the distribution of contest strength among active players. Parts (ii) and (iii) show that redistribution is governed by relative contest strength within the active set. A reduction in an active player's inefficiency raises her share and lowers the shares of other active players.

The next proposition studies the case in which an efficiency gain induces entry by a previously inactive player.

Proposition 3 (Entry of a Previously Inactive Player)**.** *Let $i \notin \mathcal{L}$ be initially inactive. Suppose a reduction in her inefficiency, $a_i \mapsto a_i - \varepsilon$, makes her active, and let s'_k denote player k's post-entry share. Then:*

(i) *the entrant obtains a strictly positive share, $s'_i > 0$, and this share is strictly increasing in ε;*

(ii) *every incumbent's share strictly declines: $s'_j - s_j < 0$ for all $j \in \mathcal{L}$;*

(iii) *inequality among incumbents widens: if $c_j > c_k$ for $j, k \in \mathcal{L}$, then $\frac{s'_j}{s'_k} > \frac{s_j}{s_k}$.*

The proof is in Appendix 5. The mechanism is straightforward. Entry lowers the participation cutoff and therefore reduces incumbents' contest strengths by the same absolute amount. The entrant gains a positive share, while every incumbent loses share. Because the absolute reduction is identical across incumbents, weaker incumbents suffer a larger proportional loss than stronger ones. Entry therefore compresses inequality across the extensive margin while widening inequality within the incumbent set.

This distinction is important. Participation-expanding reform broadens access and shifts redistribution toward previously excluded players, but it need not equalize outcomes among incumbents. In fact, the same reform that compresses inequality between participants and non-participants can sharpen the relative advantage of already strong incumbents. The overall distributive effect therefore has two margins: an extensive-margin compression driven by entry, and an intensive-margin widening within the incumbent set. Which margin matters more is ultimately an empirical question, but the model shows that the two effects move in opposite directions and should not be conflated.

The next result turns to uniform efficiency gains. This is the paper's main contrast: even symmetric reforms can generate unequal redistribution outcomes.

Proposition 4 (Redistribution Shares under Uniform Efficiency Gains)**.** *Consider a uniform reduction in inefficiency such that each a_i decreases by $\varepsilon > 0$. Then no inactive player becomes active. For each $i \in \mathcal{L}$,*

$$\frac{\partial s_i}{\partial \varepsilon} = \begin{cases} > 0 & \text{if } c_i > \bar{c}_{\mathcal{L}}, \\ < 0 & \text{if } c_i < \bar{c}_{\mathcal{L}}, \\ = 0 & \text{if } c_i = \bar{c}_{\mathcal{L}}, \end{cases}$$

where $\bar{c}_{\mathcal{L}}$ is the average contest strength among active players.

The proof is in Appendix 5. A uniform efficiency gain lowers the participation cutoff by $L\varepsilon/(L-1)$. Since this exceeds the direct gain ε received by any one inactive player, no inactive player enters. Among active players, each player's contest strength falls by the same absolute amount, namely $\varepsilon/(L-1)$. This equal compression has unequal proportional effects: stronger players lose relatively little, whereas weaker players lose relatively more. Redistribution therefore shifts toward incumbents with above-average contest strength.

To rule out incumbent exit after the reform, it is sufficient to assume $a_i < \tilde{a} - \varepsilon/(L-1)$ for all $i \in \mathcal{L}$. Under this condition, the active set remains unchanged and the comparative statics are local. The result does not rely on the convexity of shares in relative strength, although convexity amplifies the magnitude of the reallocation.

2.2.2 Aggregate Dissipation

I now study how the same efficiency reforms affect aggregate dissipation. Recall from Corollary 1 that the dissipation factor δ is inversely related to the concentration of contest strength within the active set: when contest strength becomes more evenly distributed, δ rises; when it becomes more concentrated, δ falls.

As before, I consider three types of efficiency gains: targeted gains that leave the active set unchanged, gains that induce entry by previously inactive players, and uniform gains that apply to all active players. Table 2 summarizes the results.

Table 2: Direction of aggregate dissipation under different efficiency interventions

Reform Type	**Feasible Change in $\mathcal{D}$**	**Condition for $\mathcal{D}\downarrow$**
(i) Targeted gain to inactives (no entry)	0	N/A
(ii) Targeted gain to actives	$\downarrow$ or $\uparrow$	Targeted active player is sufficiently strong
(iii) Entry of an inactive player	$\downarrow$ or $\uparrow$	Entrant becomes sufficiently efficient
(iv) Uniform gain	$\downarrow$	Strict unless $a_k = a_\ell$ for all $k, \ell \in \mathcal{L}$

The table highlights the key trade-off. Targeted gains to inactive players have no effect unless they induce entry. Targeted gains to active players can either increase or decrease dissipation, depending on whether they equalize or concentrate contest strength. Entry can reduce dissipation if the entrant is sufficiently strong. Uniform reform, by contrast, lowers dissipation by concentrating contest strength even as it worsens distributional outcomes.

I begin with the comparative statics of aggregate dissipation with respect to individual inefficiency, holding participation fixed. Thus assume $a_i \neq \tilde{a}$ for all $i \in \mathcal{N}$.

Proposition 5 (Individual Efficiency and Aggregate Dissipation)**.** *Suppose $a_i \neq \tilde{a}$ for all $i \in \mathcal{N}$. Then:*

(i) Inactive players are irrelevant off margin. For any player $j \notin \mathcal{L}$, $\frac{\partial \mathcal{D}}{\partial a_j} = 0$.

(ii) The effect of active-player efficiency depends on relative strength. For any player $j \in \mathcal{L}$, $\frac{\partial \mathcal{D}}{\partial a_j} > 0$ if and only if

$$\frac{c_j}{\mathcal{C}} > \frac{\delta}{L-1}. \tag{2.8}$$

The proof is in Appendix 5. Part (i) is immediate: as long as an inactive player remains off margin, changing her inefficiency does not affect the participation cutoff or the distribution of contest strength among active players. Part (ii) shows that the effect of an active player's efficiency depends on her position within the active set. Improving a strong player's efficiency increases concentration and lowers dissipation; improving a weak player's efficiency equalizes contest strength and raises dissipation.

Remark 3 (*Targeted efficiency gains and trade-offs*)*.* Targeted gains to weak active players reduce inequality within the active set but raise aggregate dissipation. Targeted gains to strong active players have the opposite effect: they lower dissipation but widen inequality.

I next consider participation-expanding reforms. Let a previously inactive player j lie initially at the participation threshold, $a_j = \tilde{a}$, and suppose her inefficiency falls to $a_j - \varepsilon$ with $\varepsilon > 0$.

Proposition 6 (Effective Entry Reduces Aggregate Dissipation)**.** *Suppose an inactive player j initially satisfies $a_j = \tilde{a}$, and her inefficiency is reduced by $\varepsilon > 0$. Then:*

(i) j becomes active for every $\varepsilon > 0$;

(ii) post-entry aggregate dissipation is lower than pre-entry aggregate dissipation if and only if

$$\varepsilon > A\bar{c}_{\mathcal{L}}\,\delta, \tag{2.9}$$

where δ is the pre-entry dissipation factor, $\bar{c}_{\mathcal{L}}$ is average contest strength among incumbents, and

$$A := \frac{2L^2}{L(L-1)+\delta}.$$

When (2.9) holds, $\mathcal{D}$ decreases monotonically in ε.

The proof is in Appendix 5. Entry has two opposing effects on dissipation. On the one hand, an additional participant intensifies competition and tends to raise total effort. On the other hand, entry lowers the participation cutoff and reduces incumbents' contest strengths by the same absolute amount, which weakens marginal incumbents disproportionately. If the entrant is sufficiently strong, or if pre-entry contest strength is already highly concentrated, the second force dominates and aggregate dissipation falls.

Condition (2.9) makes these forces precise. Entry is more likely to reduce dissipation when average incumbent strength $\bar{c}_{\mathcal{L}}$ is low or when pre-entry dissipation δ is low, that is, when contest strength is already concentrated among a small number of dominant players. The coefficient $A = \frac{2L^2}{L(L-1)+\delta}$ captures a size effect: holding δ fixed, A falls with L, so a smaller entrant gain is required to reduce dissipation in larger active sets.

Remark 4 (*No equity–efficiency trade-off under effective entry*). Taken together with Proposition 3, Proposition 6 implies that sufficiently strong participation-expanding reforms can reduce both inequality between active and inactive players and aggregate dissipation. Such reforms are therefore most effective when redistribution is initially dominated by a small group of incumbents.

Finally, I turn to uniform efficiency gains.

Proposition 7 (Uniform Efficiency Gains Reduce Aggregate Dissipation)**.** *Suppose each active player experiences a uniform efficiency gain of magnitude $\varepsilon > 0$. Then $\frac{d\mathcal{D}}{d\varepsilon} \leq 0$, with equality if and only if the contest is perfectly symmetric, that is, $a_k = a_\ell$ for all $k, \ell \in \mathcal{L}$.*

The proof is in Appendix 5. By Proposition 4, a uniform gain does not induce entry and, locally, leaves the active set unchanged. It instead reduces every active player's contest strength by the same absolute amount. Since stronger players lose proportionally less, relative contest strength becomes more concentrated, and aggregate dissipation falls. Uniform reform therefore lowers dissipation precisely by sharpening the asymmetries that increase inequality.

Remark 5. Participation-expanding reforms can reduce both inequality and dissipation, whereas uniform reforms generate the opposite mix: they increase inequality but reduce dissipation.

3 Robustness

The threshold-shifting mechanism does not rely on the linear-lottery benchmark. It arises more generally in share contests in which effort determines relative claims on a fixed surplus and participation is pinned down by a Kuhn–Tucker condition at zero effort. The benchmark is useful because it yields closed-form expressions, but the participation logic is more general: the marginal return to entry falls with aggregate contest intensity.

General environment. Suppose agents choose effort $e_i \geq 0$ and obtain utility

$$U_i(e) = (1-\phi)w_i + P\,p_i(e) - a_i\,h(e_i), \qquad p_i(e) = \frac{f(e_i)}{\sum_{k=1}^{N} f(e_k)}, \tag{3.1}$$

where $P = \phi S_W + V > 0$ is the redistributive surplus and $a_i > 0$ indexes inefficiency. Assume f and h are twice continuously differentiable, with $f(0) = 0$, $f'(e) > 0$, $f''(e) \leq 0$, $h(0) = 0$, $h'(e) > 0$, and $h''(e) \geq 0$. To keep the entry margin nondegenerate, assume $f'(0) \in (0, \infty)$ and $h'(0) \in (0, \infty)$.[4]

Let $F(e) := \sum_{k=1}^{N} f(e_k)$ and $F_{-i}(e) := \sum_{k \neq i} f(e_k)$. For $F(e) > 0$,

$$\frac{\partial p_i(e)}{\partial e_i} = \frac{f'(e_i)\,F_{-i}(e)}{F(e)^2}. \tag{3.2}$$

Hence, for any active player i, the interior first-order condition is

$$P \cdot \frac{f'(e_i)\,F_{-i}(e)}{F(e)^2} = a_i\,h'(e_i). \tag{3.3}$$

Participation cutoff from the zero-effort margin. Fix opponents' effort profile with $F_{-i} > 0$. Evaluating (3.2) at $e_i = 0$ gives $\frac{\partial p_i}{\partial e_i}\Big|_{e_i=0} = f'(0)/F_{-i}$. The Kuhn–Tucker condition at zero effort is therefore

$$P \cdot \frac{f'(0)}{F_{-i}} \leq a_i\,h'(0). \tag{3.4}$$

This is the core scaling relation: the marginal benefit of entry is proportional to $1/F_{-i}$. As aggregate contest intensity rises, a marginal unit of effort has a smaller effect on a player's share, so entry becomes harder even if all agents face lower absolute costs.

Rearranging (3.4) yields the cutoff rule $e_i > 0 \iff a_i < \tilde{a}(F_{-i})$, where

$$\tilde{a}(F) := \frac{P\,f'(0)}{h'(0)} \cdot \frac{1}{F}. \tag{3.5}$$

Equilibrium cutoff. Let $L = \{i : e_i^* > 0\}$ be the equilibrium active set and let $F^* := \sum_{k \in L} f(e_k^*)$. For the marginal participant, the relevant opponent intensity is F^*, so the equilibrium cutoff satisfies

$$\tilde{a}^* = \frac{P\,f'(0)}{h'(0)} \cdot \frac{1}{F^*}, \qquad e_i^* > 0 \iff a_i < \tilde{a}^*. \tag{3.6}$$

The benchmark lottery model is a special case in which F^* and $\tilde{a}^*$ can be written in closed form. The robustness argument relies only on (3.6).

[4] If $h'(0) = 0$ or $f'(0) = \infty$, the Kuhn–Tucker condition at $e = 0$ becomes degenerate and the participation margin may cease to be informative.

Uniform efficiency reforms. Consider a uniform reform $a_i(\varepsilon) = a_i - \varepsilon$, and work locally on a region in which the active set does not change. Under standard regularity conditions ensuring a locally unique, differentiable, and stable interior equilibrium on the active set,[5] the implicit function theorem implies $de_i^*/d\varepsilon \geq 0$ for all $i \in L$. Since $f'(e_i^*) > 0$, aggregate contest intensity rises:

$$\frac{dF^*}{d\varepsilon} = \sum_{i \in L} f'(e_i^*) \, \frac{de_i^*}{d\varepsilon} > 0. \tag{3.7}$$

Differentiating (3.6) then gives

$$\frac{d\tilde{a}^*}{d\varepsilon} = -\frac{\tilde{a}^*}{F^*} \frac{dF^*}{d\varepsilon} < 0. \tag{3.8}$$

Thus, even though all agents become more efficient, equilibrium intensity rises and the participation cutoff tightens.

When participation contracts. After the reform, the entry condition becomes $a_i - \varepsilon < \tilde{a}^*(\varepsilon)$, or equivalently $a_i < \tilde{a}^*(\varepsilon) + \varepsilon$. Define $\theta(\varepsilon) := \tilde{a}^*(\varepsilon) + \varepsilon$. Participation expands if θ rises and contracts if θ falls. Using (3.8),

$$\frac{d\theta}{d\varepsilon} = 1 + \frac{d\tilde{a}^*}{d\varepsilon} = 1 - \frac{\tilde{a}^*}{F^*} \frac{dF^*}{d\varepsilon}. \tag{3.9}$$

Hence participation contracts if and only if

$$\frac{\tilde{a}^*}{F^*} \frac{dF^*}{d\varepsilon} > 1. \tag{3.10}$$

This is the general counterpart of the benchmark result: uniform cost reductions make entry easier directly, but they also intensify competition and tighten the participation cutoff. Participation shrinks exactly when the second force dominates the first.

Role of convex costs. Convex costs strengthen this channel because they tilt the intensity response toward initially efficient players. A useful summary statistic is the cost-weighted mean inefficiency among active agents,

$$\bar{a}_h := \frac{\sum_{i \in L} a_i \, h'(e_i^*)}{\sum_{i \in L} h'(e_i^*)}.$$

[5] A sufficient condition is Rosen's diagonally strict concavity for the active-set game, together with strategic substitutability; see Rosen (1965) and Vives (1999).

Differentiating yields

$$\frac{d\bar{a}_h}{d\varepsilon} = -1 + \frac{\sum_{i\in L}(a_i - \bar{a}_h)\, h''(e_i^*)\, \frac{de_i^*}{d\varepsilon}}{\sum_{i\in L} h'(e_i^*)}. \tag{3.11}$$

The first term is mechanical. The second captures reweighting: when $h'' > 0$, effort expansions by initially efficient players receive greater marginal weight, so the effective composition of the active set improves more quickly. Through (3.7)–(3.8), this accelerates the tightening of $\tilde{a}^*$ and makes condition (3.10) more likely to hold.

Takeaway. The benchmark mechanism therefore extends to a broad class of share contests with concave contest technologies and convex effort costs. Participation is pinned down by a zero-effort Kuhn–Tucker condition whose marginal benefit scales as $1/F^*$. Uniform efficiency reforms raise equilibrium intensity under standard regularity conditions, which tightens the entry cutoff mechanically. Participation contracts whenever this endogenous tightening dominates the direct reduction in inefficiency, and convex costs strengthen the effect by shifting the intensity response toward the most efficient active agents.

This section establishes the robustness of the threshold-shifting mechanism. It does not claim that all benchmark distributional results carry over unchanged to every contest success function. The claim is narrower: the core participation logic survives beyond the linear-lottery case.

3.1 Further Discussion: Wealth–Efficiency Alignment

The benchmark model abstracts from wealth on the cost side in order to isolate the threshold-shifting mechanism as cleanly as possible. This is most useful when pre-existing wealth inequality is modest relative to the contested surplus, or when access to redistribution depends mainly on administrative, organizational, or informational efficiency rather than on financial resources. In many applications, however, wealth may itself shape effective contest performance through lobbying capacity, information, connections, or private investments that lower the cost of effort. In such environments, the distributive effect of reform depends not only on how it changes participation, but also on how wealth is aligned with contest efficiency.

When wealth and efficiency are positively aligned, the benchmark mechanism is reinforced. Agents who are already wealthy are also the most effective competitors, so a uniform reform that shifts redistribution toward high-efficiency participants also shifts redistribution toward high-wealth participants. In that case, uniform reform is more likely to amplify wealth inequality, while participation-expanding reform is more likely to reduce it by shifting redistribution toward marginal or previously excluded agents.

When wealth and efficiency are negatively aligned, the logic is reversed. If relatively poor agents are more efficient competitors, then a reform that reallocates redistribution toward efficiency may compress wealth inequality rather than widen it. More generally, once initial wealth dispersion becomes important, the sign of the inequality effect depends on whether reform reallocates contest shares toward richer or poorer agents. The benchmark model identifies how reform changes equilibrium shares through participation and contest strength; wealth–efficiency alignment determines how those share changes map into wealth inequality.

This discussion should be read as an interpretation of the benchmark mechanism rather than as a separate formal extension. The central point is that the same reform can have different wealth-distribution effects depending on who becomes relatively stronger when frictions fall.

4 Conclusion

This paper studies how reform affects redistribution when access to a common surplus is determined through endogenous competition. In the model, players differ in contest efficiency and choose whether to exert costly effort to claim a share of the surplus. Participation is therefore selective, and redistributive outcomes depend not only on formal policy parameters but also on how reform changes the participation margin and the distribution of contest strength among active players.

The analysis yields a sharp contrast between two policy margins. Uniform efficiency gains—such as broad reductions in administrative frictions or access costs—can backfire. By tightening the endogenous participation cutoff, they do not broaden access and instead reallocate redistribution toward already strong participants. Inequality within the active set therefore rises even as aggregate dissipation falls. By contrast, participation-expanding reforms operate through the extensive margin. When they move previously excluded players across the entry threshold, they broaden access directly and can reduce both inequality and dissipation, especially when entrants become sufficiently efficient and pre-reform contest strength is highly concentrated.

The broader implication is that equal treatment in reform design need not be inclusionary in equilibrium. In environments where access depends on costly initiative, information, or bureaucratic navigation, broad efficiency-enhancing reforms may strengthen incumbents unless they are paired with measures that relax barriers at the participation margin. More generally, the paper shows that the distributive effect of reform depends not only on how much frictions fall, but also on how reform changes who is able to compete for redistribution in the first place.

The analysis is intentionally stylized. It abstracts from dynamics, learning, coalition formation, and richer institutional features in order to isolate one mechanism cleanly:

policy can change equilibrium access by shifting the endogenous participation cutoff. Extending the framework to dynamic settings, endogenous investment in contest capacity, or environments with collective organization among weaker participants would be natural directions for future work. But the central lesson is already clear in the static model: reforms that lower frictions uniformly need not broaden access, whereas reforms that directly relax the participation margin can do so and may improve both equity and efficiency.

References

Aghion, P., U. Akcigit, and P. Howitt (2015). The schumpeterian growth paradigm. *Annual review of economics 7*(1), 557–575.

Aghion, P., A. Bergeaud, T. Boppart, P. J. Klenow, and H. Li (2023). A theory of falling growth and rising rents. *Review of Economic Studies 90*(6), 2675–2702.

Aghion, P. and P. Bolton (1997). A theory of trickle-down growth and development. *The review of economic studies 64*(2), 151–172.

Andrews, C., A. de Montesquiou, I. A. Sánchez, P. V. Dutta, B. V. Paul, S. Samaranayake, J. Heisey, T. Clay, and S. Chaudhary (2021). *The state of economic inclusion report 2021: The potential to scale.* World Bank Publications.

Banerjee, A., E. Breza, E. Duflo, and C. Kinnan (2019). Can microfinance unlock a poverty trap for some entrepreneurs? Technical report, National Bureau of Economic Research.

Banerjee, A., E. Duflo, R. Glennerster, and C. Kinnan (2015). The miracle of microfinance? evidence from a randomized evaluation. *American economic journal: Applied economics 7*(1), 22–53.

Banerjee, A., E. Duflo, C. Imbert, S. Mathew, and R. Pande (2020). E-governance, accountability, and leakage in public programs: Experimental evidence from a financial management reform in india. *American Economic Journal: Applied Economics 12*(4), 39–72.

Bardhan, P. and D. Mookherjee (2000). Capture and governance at local and national levels. *American economic review 90*(2), 135–139.

Bardhan, P. and D. Mookherjee (2006). Pro-poor targeting and accountability of local governments in west bengal. *Journal of development Economics 79*(2), 303–327.

Björkman, M. and J. Svensson (2009). Power to the people: evidence from a randomized field experiment on community-based monitoring in uganda. *The Quarterly Journal of Economics 124*(2), 735–769.

Casey, K., R. Glennerster, and E. Miguel (2012). Reshaping institutions: Evidence on aid impacts using a preanalysis plan. *The Quarterly Journal of Economics 127*(4), 1755–1812.

Daminato, C. and L. Pistaferri (2024). Returns heterogeneity and consumption inequality over the life cycle. Technical report, National Bureau of Economic Research.

Dutta, P., R. Murgai, M. Ravallion, and D. Van de Walle (2014). *Right to work?: assessing India's employment guarantee scheme in Bihar*. World Bank Publications.

Esteban, J. and D. Ray (1999). Conflict and distribution. *Journal of Economic Theory 87*(2), 379–415.

Esteban, J. and D. Ray (2011). Linking conflict to inequality and polarization. *American Economic Review 101*(4), 1345–1374.

Fonseca, J. and A. Matray (2024). Financial inclusion, economic development, and inequality: Evidence from brazil. *Journal of Financial Economics 156*, 103854.

Galor, O. and O. Moav (2004). From physical to human capital accumulation: Inequality and the process of development. *The review of economic studies 71*(4), 1001–1026.

Gradstein, M. (1995). Intensity of competition, entry and entry deterrence in rent seeking contests. *Economics & Politics 7*(1), 79–91.

Impullitti, G. and P. Rendahl (2025). Market power, growth, and wealth inequality. Technical report, CITP Working Paper.

Konrad, K. A. (2009). *Strategy and dynamics in contests*. Oxford University Press.

Moldovanu, B. and A. Sela (2006). Contest architecture. *Journal of Economic Theory 126*(1), 70–96.

Muralidharan, K., P. Niehaus, and S. Sukhtankar (2016). Building state capacity: Evidence from biometric smartcards in india. *American Economic Review 106*(10), 2895–2929.

Nti, K. O. (1999). Rent-seeking with asymmetric valuations. *Public Choice 98*(3), 415–430.

Olken, B. A. (2007). Monitoring corruption: evidence from a field experiment in indonesia. *Journal of political Economy 115*(2), 200–249.

Olken, B. A. and R. Pande (2012). Corruption in developing countries. *Annu. Rev. Econ. 4*(1), 479–509.

Reinikka, R. and J. Svensson (2004). Local capture: evidence from a central government transfer program in uganda. *The quarterly journal of economics 119*(2), 679–705.

Skaperdas, S. (1996). Contest success functions. *Economic theory 7*(2), 283–290.

Tu, Y., B. Chen, C. Liao, S. Wu, J. An, C. Lin, P. Gong, B. Chen, H. Wei, and B. Xu (2025). Inequality in infrastructure access and its association with health disparities. *Nature Human Behaviour*, 1–14.

Tullock, G. (1980). Efficient rent seeking. In J. M. Buchanan, R. D. Tollison, and G. Tullock (Eds.), *Toward a Theory of the Rent-Seeking Society*, pp. 97–112. Texas A&M University Press.

5 Appendix

Proof of Proposition 1. **Step 1 (Individual optimization).** Fix player $i \in \mathcal{N}$. Taking opponents' effort $E_{-i} := \sum_{j \neq i} e_j$ as given, with total effort $E_N = e_i + E_{-i}$, player i chooses $e_i \geq 0$ to maximize utility in (2.2). Using $P := \phi S_W + V$, that problem can be written as

$$\max_{e_i \geq 0} \left\{ (1-\phi) w_i + \frac{P\, e_i}{e_i + E_{-i}} - a_i e_i \right\}, \tag{5.1}$$

where $a_i = \kappa_i + \rho$. This follows from substituting $R_i = w_i - \rho e_i$ into (2.2), which yields $U_i = (1-\phi)w_i + \frac{e_i}{E_N}(\phi S_W + V) - (\kappa_i + \rho) e_i$.

For $P > 0$, the objective in (5.1) is strictly concave in e_i, since $\frac{\partial^2 U_i}{\partial e_i^2} = -\frac{2PE_{-i}}{(e_i + E_{-i})^3} < 0$ whenever $E_{-i} > 0$. Hence the Kuhn–Tucker conditions are sufficient. They are $\frac{P(E_N - e_i^*)}{E_N^2} - a_i \leq 0$, $e_i^* \geq 0$, and $e_i^* \left(\frac{P(E_N - e_i^*)}{E_N^2} - a_i \right) = 0$. Whenever $e_i^* > 0$, complementary slackness implies $\frac{P(E_N - e_i^*)}{E_N^2} = a_i$.

Step 2 (Precluding trivial equilibria). Any pure-strategy Nash equilibrium must involve at least two active players.

If all players choose zero effort, then by convention each receives an equal share of the contested surplus. A unilateral deviation by any player i to a small effort $e_i = \varepsilon > 0$ yields $U_i(\varepsilon) = (1-\phi)w_i + P - a_i \varepsilon$, which exceeds $(1-\phi)w_i + P/N$ for sufficiently small ε. Hence $E_N = 0$ cannot be an equilibrium.

If exactly one player i is active, then $E_N = e_i$ and $U_i(e_i) = (1-\phi)w_i + P - a_i e_i$. Since the prize P is then independent of e_i, player i can profitably reduce effort and lower cost without affecting the payoff. Hence no equilibrium can have exactly one active player.

Therefore, any equilibrium must feature an active set $\mathcal{L} \subseteq \mathcal{N}$ with $L := |\mathcal{L}| \geq 2$.

Step 3 (Equilibrium effort and participation). For any active player $i \in \mathcal{L}$, the first-order condition holds with equality, so $P(E_N - e_i) = a_i E_N^2$. Summing over all $i \in \mathcal{L}$ gives $P(L-1)E_N = E_N^2 \sum_{j\in\mathcal{L}} a_j$, and therefore $E_N^* = \frac{P(L-1)}{\sum_{j\in\mathcal{L}} a_j}$. Define the participation threshold by $\tilde{a} := \frac{1}{L-1}\sum_{j\in\mathcal{L}} a_j$. Then $E_N^* = P/\tilde{a}$. Substituting this into the first-order condition yields $e_i^* = \frac{P(\tilde{a}-a_i)}{\tilde{a}^2}$ for $i \in \mathcal{L}$, and $e_j^* = 0$ for $j \notin \mathcal{L}$. Hence $e_i^* > 0$ if and only if $a_i < \tilde{a}$, so the active set is exactly $\mathcal{L} = \{i \in \mathcal{N} : a_i < \tilde{a}\}$. By Lemma 1, the threshold $\tilde{a}$ and active set $\mathcal{L}$ are unique. Given $\mathcal{L}$, strict concavity of each player's problem implies a unique equilibrium effort vector.

Step 4 (Equilibrium utility). For any inactive player $j \notin \mathcal{L}$, equilibrium utility is $U_j^* = (1-\phi)w_j$. For any active player $i \in \mathcal{L}$, substituting the equilibrium effort into (5.1) yields $U_i^* = (1-\phi)w_i + P\left(\frac{e_i^*}{E_N^*}\right)^2$. Now $\frac{e_i^*}{E_N^*} = \frac{\tilde{a}-a_i}{\tilde{a}}$. Define $c_i := \max\{\tilde{a} - a_i, 0\}$ and $\mathcal{C} := \sum_{j\in\mathcal{L}} c_j$. Since $\mathcal{C} = \sum_{j\in\mathcal{L}}(\tilde{a} - a_j) = L\tilde{a} - \sum_{j\in\mathcal{L}} a_j = L\tilde{a} - (L-1)\tilde{a} = \tilde{a}$, it follows that $\frac{e_i^*}{E_N^*} = \frac{c_i}{\mathcal{C}}$. Therefore the equilibrium redistribution share is $s_i := \left(\frac{c_i}{\mathcal{C}}\right)^2$, and $U_i^* = (1-\phi)w_i + s_i(\phi S_W + V)$ for all $i \in \mathcal{N}$. □

Proof of Corollary 1. Let $P := \phi S_W + V$. Aggregate dissipation is the total equilibrium effort cost incurred by active players: $\mathcal{D} = \sum_{i\in\mathcal{L}} a_i e_i^*$. By Proposition 1, every active player satisfies the first-order condition $a_i = P(E_N^* - e_i^*)/(E_N^*)^2$. Substituting gives

$$\begin{aligned}
\mathcal{D} &= \sum_{i\in\mathcal{L}} \left(P\frac{E_N^* - e_i^*}{(E_N^*)^2}\right) e_i^* \\
&= \frac{P}{(E_N^*)^2}\sum_{i\in\mathcal{L}}\Big(E_N^* e_i^* - (e_i^*)^2\Big) \\
&= \frac{P}{(E_N^*)^2}\left(E_N^*\sum_{i\in\mathcal{L}} e_i^* - \sum_{i\in\mathcal{L}}(e_i^*)^2\right).
\end{aligned}$$

Since $E_N^* = \sum_{i\in\mathcal{L}} e_i^*$, this simplifies to $\mathcal{D} = P\left(1 - \frac{\sum_{i\in\mathcal{L}}(e_i^*)^2}{(E_N^*)^2}\right)$.

Again by Proposition 1, equilibrium effort is proportional to contest strength: $e_i^* = kc_i$, where $k = P/\tilde{a}^2$ and $c_i = \tilde{a} - a_i$ for $i \in \mathcal{L}$. Hence $\frac{\sum_{i\in\mathcal{L}}(e_i^*)^2}{(E_N^*)^2} = \frac{\sum_{i\in\mathcal{L}}(kc_i)^2}{\left(\sum_{j\in\mathcal{L}} kc_j\right)^2} = \frac{\sum_{i\in\mathcal{L}} c_i^2}{\left(\sum_{j\in\mathcal{L}} c_j\right)^2} =$ HHI(c). Therefore

$$\mathcal{D} = P\left[1 - \text{HHI}(c)\right] = \delta(\phi S_W + V),$$

where $\delta := 1 - \text{HHI}(c)$. This proves (2.6). □

Proof of Proposition 2. Throughout, the active set is held fixed.

(i) Inactive players. Let $k \notin \mathcal{L}$. As long as the perturbation does not induce entry, that is, as long as $a_k > \tilde{a}$, the participation threshold $\tilde{a}$, the contest strengths $\{c_i\}_{i\in\mathcal{L}}$, and total contest strength $\mathcal{C}$ remain unchanged. Hence $s_i = (c_i/\mathcal{C})^2$ is unaffected for every $i \in \mathcal{N}$, so $\frac{\partial s_i}{\partial a_k} = 0$.

(ii) Own inefficiency. Let $i \in \mathcal{L}$. Since $s_i = (c_i/\mathcal{C})^2$, differentiation gives

$$\frac{\partial s_i}{\partial a_i} = \frac{2c_i}{\mathcal{C}^3}\left(\frac{\partial c_i}{\partial a_i}\mathcal{C} - c_i\frac{\partial \mathcal{C}}{\partial a_i}\right).$$

Because $\tilde{a} = \frac{1}{L-1}\sum_{j\in\mathcal{L}} a_j$, I have $\frac{\partial \tilde{a}}{\partial a_i} = \frac{1}{L-1}$. Therefore $\frac{\partial c_i}{\partial a_i} = \frac{\partial \tilde{a}}{\partial a_i} - 1 = -\frac{L-2}{L-1}$ and $\frac{\partial \mathcal{C}}{\partial a_i} = \frac{\partial \tilde{a}}{\partial a_i} = \frac{1}{L-1}$, where the second identity uses $\mathcal{C} = \tilde{a}$. Substituting yields $\frac{\partial s_i}{\partial a_i} = -\frac{2c_i}{(L-1)\mathcal{C}^3}\Big((L-2)\mathcal{C} + c_i\Big)$. Since $L \geq 2$, $c_i > 0$, and $\mathcal{C} > 0$, the bracketed term is strictly positive. Hence $\frac{\partial s_i}{\partial a_i} < 0$.

(iii) Cross effects. Let $j \in \mathcal{L}$ with $j \neq i$. Again,

$$\frac{\partial s_i}{\partial a_j} = \frac{2c_i}{\mathcal{C}^3}\left(\frac{\partial c_i}{\partial a_j}\mathcal{C} - c_i\frac{\partial \mathcal{C}}{\partial a_j}\right).$$

Now $\frac{\partial c_i}{\partial a_j} = \frac{\partial \tilde{a}}{\partial a_j} = \frac{1}{L-1}$ and, since $\mathcal{C} = \tilde{a}$, also $\frac{\partial \mathcal{C}}{\partial a_j} = \frac{1}{L-1}$. Therefore $\frac{\partial s_i}{\partial a_j} = \frac{2c_i}{(L-1)\mathcal{C}^3}(\mathcal{C} - c_i)$. For any active player i, one has $0 < c_i < \mathcal{C}$ when $L \geq 2$. Hence $\mathcal{C} - c_i > 0$, so $\frac{\partial s_i}{\partial a_j} > 0$.

Thus, off-margin inactive players are irrelevant, an increase in an active player's own inefficiency lowers her share, and an increase in another active player's inefficiency raises it. □

Proof of Proposition 3. Let the initial active set be $\mathcal{L}$ with $|\mathcal{L}| = L$ and participation threshold $\tilde{a}$. Consider a marginally inactive player i with $a_i = \tilde{a}$.[6] Suppose her inefficiency falls to $a_i' = \tilde{a} - \varepsilon$ for some $\varepsilon > 0$.

Assume ε is small enough that no incumbent exits. A sufficient condition is $\varepsilon < L\Big(\tilde{a} - \max_{j\in\mathcal{L}} a_j\Big)$, which ensures $a_j < \tilde{a} - \varepsilon/L$ for all $j \in \mathcal{L}$. Then the post-reform active set is $\mathcal{L}' = \mathcal{L} \cup \{i\}$, with size $L + 1$.

Step 1 (Post-entry cutoff and contest strengths). Let $A := \sum_{j\in\mathcal{L}} a_j$, so $\tilde{a} = A/(L-1)$. After entry, the new threshold is $\tilde{a}' = \frac{A+a_i'}{L} = \frac{(L-1)\tilde{a}+(\tilde{a}-\varepsilon)}{L} = \tilde{a} - \frac{\varepsilon}{L}$. Since total contest strength equals the threshold in equilibrium, $\mathcal{C}' = \tilde{a}'$.

The entrant's contest strength is $c_i' = \tilde{a}' - a_i' = \left(\tilde{a} - \frac{\varepsilon}{L}\right) - (\tilde{a} - \varepsilon) = \varepsilon\left(1 - \frac{1}{L}\right) > 0$. For each incumbent $j \in \mathcal{L}$, $c_j' = \tilde{a}' - a_j = c_j - \frac{\varepsilon}{L}$. Thus entry reduces every incumbent's contest strength by the same absolute amount, ε/L.

Step 2 (Entrant's share). Because $c_i' > 0$, the entrant receives a strictly positive redistribution share,

$$s_i' = \left(\frac{c_i'}{\mathcal{C}'}\right)^2 = \left(\frac{\varepsilon(1 - 1/L)}{\tilde{a} - \varepsilon/L}\right)^2 > 0.$$

Moreover, s_i' is strictly increasing in ε, since both the numerator and the ratio itself increase with ε on the admissible range.

[6] Equivalently, one may start from $a_i > \tilde{a}$ but arbitrarily close to $\tilde{a}$; the same limiting argument applies.

Step 3 (Incumbents' shares). For any incumbent $j \in \mathcal{L}$, $s'_j = \left(\frac{c_j-\varepsilon/L}{\tilde{a}-\varepsilon/L}\right)^2$. To compare this with $s_j = (c_j/\tilde{a})^2$, note that $\frac{c_j-\varepsilon/L}{\tilde{a}-\varepsilon/L} < \frac{c_j}{\tilde{a}}$ if and only if $\tilde{a}\left(c_j - \frac{\varepsilon}{L}\right) < c_j\left(\tilde{a} - \frac{\varepsilon}{L}\right)$, which is equivalent to $\tilde{a} > c_j$. This inequality holds for every active incumbent, since $c_j = \tilde{a} - a_j < \tilde{a}$. Hence $s'_j < s_j$ for all $j \in \mathcal{L}$.

Step 4 (Inequality among incumbents). Take any $j, k \in \mathcal{L}$ with $c_j > c_k$. Then $\frac{s'_j}{s'_k} = \left(\frac{c_j-\varepsilon/L}{c_k-\varepsilon/L}\right)^2$. Because $c_j > c_k > \varepsilon/L$, the ratio $(c_j - \varepsilon/L)/(c_k - \varepsilon/L)$ is strictly increasing in ε. Therefore $\frac{s'_j}{s'_k} > \frac{s_j}{s_k}$. So although all incumbents lose in levels, stronger incumbents lose proportionally less than weaker incumbents. Entry therefore widens inequality within the incumbent set.

This proves all three claims. □

Proof of Proposition 4. Consider a uniform reduction in inefficiency of size $\varepsilon > 0$, so that $a'_k = a_k - \varepsilon$ for every $k \in \mathcal{N}$.

Step 1 (Participation). Suppose first that $j \notin \mathcal{L}$. Under the uniform reform, j becomes active only if $a'_j < \tilde{a}'$. Since the active set is initially $\mathcal{L}$, the post-reform cutoff associated with the same active set is $\tilde{a}' = \frac{1}{L-1}\sum_{i\in\mathcal{L}} a'_i = \frac{1}{L-1}\sum_{i\in\mathcal{L}}(a_i - \varepsilon) = \tilde{a} - \frac{L\varepsilon}{L-1}$. Hence entry requires $a_j - \varepsilon < \tilde{a} - \frac{L\varepsilon}{L-1}$, or equivalently, $a_j - \tilde{a} < -\frac{\varepsilon}{L-1}$. But every inactive player satisfies $a_j \geq \tilde{a}$, so this inequality cannot hold. Thus no inactive player enters.

Now consider $i \in \mathcal{L}$. Player i exits only if $a'_i \geq \tilde{a}'$, that is, $a_i - \varepsilon \geq \tilde{a} - \frac{L\varepsilon}{L-1}$, or equivalently, $a_i - \tilde{a} \geq -\frac{\varepsilon}{L-1}$. Since $a_i < \tilde{a}$ for all $i \in \mathcal{L}$, this condition fails for sufficiently small ε. Therefore, locally, the active set remains unchanged.

Step 2 (Contest strengths). For each active player $i \in \mathcal{L}$, the post-reform contest strength is $c'_i = \tilde{a}' - a'_i = \left(\tilde{a} - \frac{L\varepsilon}{L-1}\right) - (a_i - \varepsilon) = c_i - \frac{\varepsilon}{L-1}$. Summing across active players gives $\mathcal{C}' = \sum_{i\in\mathcal{L}} c'_i = \mathcal{C} - \frac{L\varepsilon}{L-1}$.

Step 3 (Redistribution shares). For each $i \in \mathcal{L}$, the post-reform redistribution share is $s'_i = (c'_i/\mathcal{C}')^2$. Differentiating with respect to ε and evaluating at $\varepsilon = 0$ gives

$$\frac{\partial s_i}{\partial \varepsilon} = \frac{2c_i}{\mathcal{C}^3}\left(\mathcal{C}\frac{\partial c'_i}{\partial \varepsilon} - c_i\frac{\partial \mathcal{C}'}{\partial \varepsilon}\right).$$

Since $\frac{\partial c'_i}{\partial \varepsilon} = -\frac{1}{L-1}$ and $\frac{\partial \mathcal{C}'}{\partial \varepsilon} = -\frac{L}{L-1}$, it follows that $\frac{\partial s_i}{\partial \varepsilon} = -\frac{2c_i}{(L-1)\mathcal{C}^3}(\mathcal{C} - Lc_i)$. Let $\bar{c}_{\mathcal{L}} := \mathcal{C}/L$ denote average contest strength among active players. Then $\mathcal{C} - Lc_i = L(\bar{c}_{\mathcal{L}} - c_i)$, so

$$\frac{\partial s_i}{\partial \varepsilon} = -\frac{2Lc_i}{(L-1)\mathcal{C}^3}(\bar{c}_{\mathcal{L}} - c_i).$$

Because the prefactor is strictly positive, the sign of $\frac{\partial s_i}{\partial \varepsilon}$ is determined by $c_i - \bar{c}_{\mathcal{L}}$. Thus $\frac{\partial s_i}{\partial \varepsilon} > 0$ if $c_i > \bar{c}_{\mathcal{L}}$, $\frac{\partial s_i}{\partial \varepsilon} < 0$ if $c_i < \bar{c}_{\mathcal{L}}$, and $\frac{\partial s_i}{\partial \varepsilon} = 0$ if $c_i = \bar{c}_{\mathcal{L}}$.

Hence a uniform efficiency gain leaves participation unchanged locally, but reallocates redistribution toward players with above-average contest strength. □

Proof of Proposition 5. Assume throughout that perturbations are small enough that the active set $\mathcal{L}$ remains unchanged.

(i) Inactive players. Let $j \notin \mathcal{L}$. By Corollary 1, aggregate dissipation is $\mathcal{D} = P(1 - \mathrm{HHI}(c))$, where $P = \phi S_W + V$ and $\mathrm{HHI}(c)$ depends only on the contest strengths of active players. As long as j remains inactive, changing a_j does not affect $\tilde{a}$, $\mathcal{L}$, or $\{c_i\}_{i\in\mathcal{L}}$. Hence $\mathrm{HHI}(c)$ is unchanged, so $\frac{\partial \mathcal{D}}{\partial a_j} = 0$.

(ii) Active players. Let $j \in \mathcal{L}$. Since $\mathcal{D} = P(1 - \mathrm{HHI}(c))$, I have $\frac{\partial \mathcal{D}}{\partial a_j} = -P\,\frac{\partial \mathrm{HHI}(c)}{\partial a_j}$. Now $\mathrm{HHI}(c) = \frac{\sum_{i\in\mathcal{L}} c_i^2}{\mathcal{C}^2}$. Differentiating by the quotient rule gives

$$\frac{\partial \mathrm{HHI}(c)}{\partial a_j} = \frac{\left(\frac{\partial}{\partial a_j}\sum_{i\in\mathcal{L}} c_i^2\right)\mathcal{C}^2 - \left(\sum_{i\in\mathcal{L}} c_i^2\right)\frac{\partial \mathcal{C}^2}{\partial a_j}}{\mathcal{C}^4}.$$

First, $\frac{\partial}{\partial a_j}\sum_{i\in\mathcal{L}} c_i^2 = 2c_j\frac{\partial c_j}{\partial a_j} + \sum_{i\in\mathcal{L},\, i\neq j} 2c_i\frac{\partial c_i}{\partial a_j}$. By Proposition 2, $\frac{\partial c_j}{\partial a_j} = -\frac{L-2}{L-1}$ and $\frac{\partial c_i}{\partial a_j} = \frac{1}{L-1}$ for $i \neq j$. Therefore $\frac{\partial}{\partial a_j}\sum_{i\in\mathcal{L}} c_i^2 = \frac{2}{L-1}\left(-(L-2)c_j + \sum_{i\neq j} c_i\right) = \frac{2}{L-1}\Big(\mathcal{C} - (L-1)c_j\Big)$.
Second, since $\mathcal{C} = \tilde{a}$, Proposition 2 gives $\frac{\partial \mathcal{C}}{\partial a_j} = \frac{1}{L-1}$, and hence $\frac{\partial \mathcal{C}^2}{\partial a_j} = 2\mathcal{C}\frac{\partial \mathcal{C}}{\partial a_j} = \frac{2\mathcal{C}}{L-1}$.
Substituting these into the quotient rule yields

$$\begin{aligned}\frac{\partial \mathrm{HHI}(c)}{\partial a_j} &= \frac{1}{\mathcal{C}^4}\left[\frac{2}{L-1}\Big(\mathcal{C} - (L-1)c_j\Big)\mathcal{C}^2 - \left(\sum_{i\in\mathcal{L}} c_i^2\right)\frac{2\mathcal{C}}{L-1}\right] \\ &= \frac{2}{(L-1)\mathcal{C}^3}\left[\mathcal{C}^2 - (L-1)c_j\mathcal{C} - \sum_{i\in\mathcal{L}} c_i^2\right].\end{aligned}$$

Using $\sum_{i\in\mathcal{L}} c_i^2 = \mathcal{C}^2\,\mathrm{HHI}(c)$, this becomes $\frac{\partial \mathrm{HHI}(c)}{\partial a_j} = \frac{2}{(L-1)\mathcal{C}}\left[(1 - \mathrm{HHI}(c)) - (L-1)\frac{c_j}{\mathcal{C}}\right]$. Let $\delta := 1 - \mathrm{HHI}(c)$. Then $\frac{\partial \mathrm{HHI}(c)}{\partial a_j} = \frac{2}{(L-1)\mathcal{C}}\left[\delta - (L-1)\frac{c_j}{\mathcal{C}}\right]$. Therefore

$$\frac{\partial \mathcal{D}}{\partial a_j} = -P\,\frac{\partial \mathrm{HHI}(c)}{\partial a_j} = \frac{2P}{(L-1)\mathcal{C}}\left[(L-1)\frac{c_j}{\mathcal{C}} - \delta\right].$$

Since $P > 0$, $L \geq 2$, and $\mathcal{C} > 0$, the sign of $\frac{\partial \mathcal{D}}{\partial a_j}$ is determined by the bracketed term. Hence

$$\frac{\partial \mathcal{D}}{\partial a_j} > 0 \quad \Longleftrightarrow \quad \frac{c_j}{\mathcal{C}} > \frac{\delta}{L-1},$$

which proves the claim. □

Proof of Proposition 6. Let the initial active set be $\mathcal{L}$ with $|\mathcal{L}| = L$. Write $C := \sum_{i\in\mathcal{L}} c_i$ and $S_2 := \sum_{i\in\mathcal{L}} c_i^2$, so that the pre-entry Herfindahl index is $\mathrm{HHI}(0) = S_2/C^2$ and the pre-entry dissipation factor is $\delta = 1 - \mathrm{HHI}(0)$.

Suppose an inactive player at the threshold, $a_j = \tilde{a}$, receives an efficiency gain $\varepsilon > 0$. By Proposition 3, the entrant's contest strength is $c_{L+1} = \frac{L-1}{L}\varepsilon$, while each incumbent's strength falls to $c_i' = c_i - \frac{\varepsilon}{L}$. Aggregate dissipation falls if and only if the post-entry

Herfindahl index rises. Thus I require

$$\frac{\left(\frac{L-1}{L}\varepsilon\right)^2 + \sum_{i\in\mathcal{L}}\left(c_i - \frac{\varepsilon}{L}\right)^2}{\left(\frac{L-1}{L}\varepsilon + \sum_{i\in\mathcal{L}}\left(c_i - \frac{\varepsilon}{L}\right)\right)^2} > \frac{S_2}{C^2}. \tag{5.2}$$

Expanding the numerator on the left-hand side gives $\left(\frac{L-1}{L}\varepsilon\right)^2 + \sum_{i\in\mathcal{L}}\left(c_i - \frac{\varepsilon}{L}\right)^2 = \frac{L^2-L+1}{L^2}\varepsilon^2 - \frac{2C\varepsilon}{L} + S_2$, while the denominator is $\left(\frac{L-1}{L}\varepsilon + \sum_{i\in\mathcal{L}}\left(c_i - \frac{\varepsilon}{L}\right)\right)^2 = \left(C - \frac{\varepsilon}{L}\right)^2$. Since denominators are positive, (5.2) is equivalent to $\left(\frac{L^2-L+1}{L^2}\varepsilon^2 - \frac{2C\varepsilon}{L} + S_2\right)C^2 > S_2\left(C - \frac{\varepsilon}{L}\right)^2$. Expanding and simplifying yields

$$\left(\frac{C^2(L^2-L+1) - S_2}{L^2}\right)\varepsilon^2 + \left(\frac{2CS_2 - 2C^3}{L}\right)\varepsilon > 0.$$

Factoring out $\varepsilon > 0$, I obtain

$$\varepsilon\left[\left(\frac{C^2(L^2-L+1) - S_2}{L^2}\right)\varepsilon - \frac{2C}{L}(C^2 - S_2)\right] > 0.$$

Because $S_2 \leq C^2$ and $L^2 - L + 1 \geq 1$, the coefficient on ε^2 is strictly positive. Hence the inequality holds if and only if

$$\varepsilon > \frac{2CL(C^2 - S_2)}{C^2(L^2-L+1) - S_2}.$$

Now substitute $S_2 = C^2\,\mathrm{HHI}(0)$ and $\delta = 1 - \mathrm{HHI}(0)$. Since $C^2 - S_2 = C^2\delta$, the threshold becomes $\varepsilon > \frac{2CL\,C^2\delta}{C^2(L^2-L+1-\mathrm{HHI}(0))} = \frac{2CL\,\delta}{L^2-L+\delta}$. Using $C = L\bar{c}_{\mathcal{L}}$, where $\bar{c}_{\mathcal{L}}$ is average pre-entry contest strength, this can be written as

$$\varepsilon > \frac{2L^2\bar{c}_{\mathcal{L}}\delta}{L^2 - L + \delta} = \frac{2\bar{c}_{\mathcal{L}}\delta}{\frac{L-1}{L} + \frac{\delta}{L^2}} = A\bar{c}_{\mathcal{L}}\delta,$$

where $A := \frac{2}{\frac{L-1}{L} + \frac{\delta}{L^2}}$. This proves the cutoff condition in (2.9).

Finally, note that the left-hand side of (5.2) is increasing in ε once the threshold is crossed, because the quadratic term above has a positive leading coefficient. Hence, whenever $\varepsilon > A\bar{c}_{\mathcal{L}}\delta$, post-entry aggregate dissipation decreases monotonically as ε rises.

To verify the size effect, differentiate $A(L) = \frac{2}{1-\frac{1}{L}+\frac{\delta}{L^2}}$ with respect to L, holding δ fixed:

$$\frac{\partial A}{\partial L} = -\frac{2(L-2\delta)}{L^3\left(1 - \frac{1}{L} + \frac{\delta}{L^2}\right)^2}.$$

Since $L \geq 2$ and $0 < \delta < 1$, the numerator is positive, so $\frac{\partial A}{\partial L} < 0$. Thus, holding the pre-entry concentration δ fixed, the entrant efficiency gain required to reduce dissipation is smaller in larger active sets. □

Proof of Proposition 7. Consider a uniform efficiency gain of size $\varepsilon > 0$, so that each active player's inefficiency falls from a_i to $a_i' = a_i - \varepsilon$. By Proposition 4, for sufficiently small ε the active set $\mathcal{L}$ remains unchanged. The post-reform contest strengths are therefore $c_i' = \tilde{a}' - a_i' = \left(\tilde{a} - \frac{L\varepsilon}{L-1}\right) - (a_i - \varepsilon) = c_i - \frac{\varepsilon}{L-1}$, and total contest strength becomes $\mathcal{C}' = \sum_{i\in\mathcal{L}} c_i' = \mathcal{C} - \frac{L\varepsilon}{L-1}$.

Aggregate dissipation satisfies $\mathcal{D} = P(1 - \text{HHI}(c))$, where $\text{HHI}(c) = \frac{\sum_{i\in\mathcal{L}} c_i^2}{\mathcal{C}^2}$. Hence it is enough to show that $\text{HHI}(c)$ weakly increases with ε.

Differentiate $\text{HHI}(c') = \frac{\sum_{i\in\mathcal{L}}(c_i')^2}{(\mathcal{C}')^2}$ with respect to ε. Since $\frac{\partial c_i'}{\partial\varepsilon} = -\frac{1}{L-1}$ for every $i \in \mathcal{L}$ and $\frac{\partial \mathcal{C}'}{\partial\varepsilon} = -\frac{L}{L-1}$, the quotient rule gives

$$\frac{d\,\text{HHI}(c')}{d\varepsilon} = \frac{\left(\sum_{i\in\mathcal{L}} 2c_i'\frac{\partial c_i'}{\partial\varepsilon}\right)(\mathcal{C}')^2 - \left(\sum_{i\in\mathcal{L}}(c_i')^2\right)2\mathcal{C}'\frac{\partial\mathcal{C}'}{\partial\varepsilon}}{(\mathcal{C}')^4}.$$

Evaluating at $\varepsilon = 0$ yields

$$\frac{d\,\text{HHI}(c)}{d\varepsilon} = \frac{-\frac{2}{L-1}\mathcal{C}\cdot\mathcal{C}^2 + \left(\sum_{i\in\mathcal{L}} c_i^2\right)\frac{2L}{L-1}\mathcal{C}}{\mathcal{C}^4} = \frac{2}{\mathcal{C}(L-1)}\left(L\,\text{HHI}(c) - 1\right).$$

By Cauchy–Schwarz, $\text{HHI}(c) \geq 1/L$, with equality if and only if $c_i = c_j$ for all $i, j \in \mathcal{L}$. Hence $L\,\text{HHI}(c) - 1 \geq 0$, so $\frac{d\,\text{HHI}(c)}{d\varepsilon} \geq 0$. Therefore the concentration of contest strength weakly increases under a uniform efficiency gain.

Since $\mathcal{D} = P(1 - \text{HHI}(c))$, it follows that

$$\frac{d\mathcal{D}}{d\varepsilon} = -P\,\frac{d\,\text{HHI}(c)}{d\varepsilon} = -\frac{2P}{\mathcal{C}(L-1)}\left(L\,\text{HHI}(c) - 1\right) \leq 0.$$

The inequality is strict whenever contest strengths are heterogeneous, that is, whenever $\text{HHI}(c) > 1/L$, and it becomes an equality only under perfect symmetry.

Thus uniform efficiency gains weakly reduce aggregate dissipation, with strict reduction except in the symmetric case. □

Appendix A. General cutoff logic beyond the linear benchmark

This appendix shows that the threshold-shifting mechanism does not rely on the linear-lottery benchmark. The key object is the zero-effort entry condition, which implies that the participation cutoff is inversely related to aggregate contest intensity.

Consider the contest success function $\pi_i(e) = \frac{f(e_i)}{\sum_{k=1}^N f(e_k)}$, where $f(0) = 0$, $f'(e) > 0$, and $f''(e) \leq 0$, and let the cost function h satisfy $h(0) = 0$, $h'(e) > 0$, and $h''(e) \geq 0$. Player i's utility is $U_i(e) = (1-\phi)w_i + P\,\pi_i(e) - a_i\,h(e_i)$, where $P = \phi S_W + V > 0$. Let

$F(e) := \sum_{k=1}^{N} f(e_k)$ and $F_{-i}(e) := \sum_{k \neq i} f(e_k)$. For $F(e) > 0$,

$$\frac{\partial \pi_i(e)}{\partial e_i} = \frac{f'(e_i)\, F_{-i}(e)}{F(e)^2}. \tag{.3}$$

Hence, for any active player i, the interior first-order condition is

$$P \cdot \frac{f'(e_i)\, F_{-i}(e)}{F(e)^2} = a_i\, h'(e_i). \tag{.4}$$

The entry margin. Evaluate (.3) at $e_i = 0$, holding opponents' effort fixed with $F_{-i} > 0$. Since $f(0) = 0$, one has $F(e) = F_{-i}$ at $e_i = 0$, so $\frac{\partial \pi_i}{\partial e_i}\Big|_{e_i=0} = \frac{f'(0)}{F_{-i}}$. The Kuhn–Tucker condition for zero effort is therefore

$$P \cdot \frac{f'(0)}{F_{-i}} \leq a_i\, h'(0). \tag{.5}$$

Thus player i enters if and only if

$$a_i < \tilde{a}(F_{-i}), \qquad \tilde{a}(F) := \frac{P\, f'(0)}{h'(0)} \cdot \frac{1}{F}. \tag{.6}$$

Equation (.6) is the general participation cutoff. It implies that the marginal benefit of entry is inversely proportional to aggregate contest intensity.

Equilibrium cutoff. Let $L = \{i : e_i^* > 0\}$ be the equilibrium active set and define equilibrium intensity by $F^* := \sum_{k \in L} f(e_k^*)$. For the marginal participant, the relevant opponent intensity is F^*. Hence the equilibrium cutoff satisfies

$$\tilde{a}^* = \frac{P\, f'(0)}{h'(0)} \cdot \frac{1}{F^*}, \qquad e_i^* > 0 \iff a_i < \tilde{a}^*. \tag{.7}$$

The benchmark lottery model is a special case of (.7). The general mechanism is the same: stronger aggregate competition lowers the participation cutoff.

Uniform efficiency reform. Consider a uniform reform $a_i(\varepsilon) = a_i - \varepsilon$, and work locally on a region where the active set does not change. Suppose standard regularity conditions guarantee a locally unique and differentiable equilibrium on the active set, and that the contest exhibits strategic substitutability. Then a uniform efficiency gain raises equilibrium effort for active players, so equilibrium contest intensity rises:

$$\frac{dF^*}{d\varepsilon} > 0. \tag{.8}$$

Differentiating (.7) yields

$$\frac{d\tilde{a}^*}{d\varepsilon} = -\frac{\tilde{a}^*}{F^*}\,\frac{dF^*}{d\varepsilon} < 0. \tag{.9}$$

Thus, even though all players become more efficient, the participation cutoff tightens.

When participation contracts. After the reform, entry requires $a_i - \varepsilon < \tilde{a}^*(\varepsilon)$, or equivalently $a_i < \tilde{a}^*(\varepsilon) + \varepsilon$. Define $\theta(\varepsilon) := \tilde{a}^*(\varepsilon) + \varepsilon$. Then

$$\frac{d\theta}{d\varepsilon} = 1 + \frac{d\tilde{a}^*}{d\varepsilon} = 1 - \frac{\tilde{a}^*}{F^*}\,\frac{dF^*}{d\varepsilon}. \tag{.10}$$

Therefore participation contracts if and only if

$$\frac{\tilde{a}^*}{F^*}\,\frac{dF^*}{d\varepsilon} > 1. \tag{.11}$$

This is the general counterpart of the benchmark result: uniform efficiency gains make entry easier directly, but they also intensify equilibrium competition and tighten the participation cutoff. Participation shrinks precisely when the second force dominates the first.

Role of convex costs. Convex costs strengthen this mechanism because they amplify the effort response of initially efficient players. A useful summary object is the cost-weighted mean inefficiency among active players,

$$\bar{a}_h := \frac{\sum_{i\in L} a_i\, h'(e_i^*)}{\sum_{i\in L} h'(e_i^*)}.$$

Differentiating gives

$$\frac{d\bar{a}_h}{d\varepsilon} = -1 + \frac{\sum_{i\in L} h''(e_i^*)\,(a_i - \bar{a}_h)\,\frac{de_i^*}{d\varepsilon}}{\sum_{i\in L} h'(e_i^*)}. \tag{.12}$$

The first term is mechanical. The second captures reweighting: if efficient players increase effort more after the reform, convex costs place greater weight on them, so the effective composition of the active set improves more quickly. This raises F^* more sharply and strengthens the tightening of the participation cutoff in (.9).

Takeaway. The benchmark mechanism extends to a broad class of contests. The crucial object is the zero-effort Kuhn–Tucker condition, which implies that the participation cutoff is inversely related to aggregate contest intensity. Uniform efficiency reforms raise equilibrium intensity under standard regularity conditions, so the cutoff tightens mechanically. Participation contracts whenever this endogenous tightening dominates the direct reduction in inefficiency, and convex costs make that outcome more likely by tilting the effort response toward the most efficient active players.