When Uniform Access Reforms Backfire: Endogenous Participation in Contests[1]

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Abstract

Many access reforms are designed to broaden participation by lowering the cost of becoming an effective competitor. This paper shows that such reforms can have the opposite effect when access is contestable. I study a contestable-access model with ratio-form payoffs in which potential claimants differ in the cost of producing effective contest intensity. A uniform reform helps marginal claimants by lowering their cost of becoming effective competitors, but it also strengthens low-cost incumbents. When incumbents respond strongly, aggregate contest intensity rises and the endogenous participation cutoff moves against marginal claimants. Participation then falls even though every claimant faces a lower cost. I characterize this reversal using a lower-tail concentration index that measures whether effective intensity is concentrated among the lowest-cost active claimants or spread toward the participation margin. A uniform reform expands participation when intensity is spread toward the margin and contracts participation when intensity is concentrated among low-cost incumbents. I then derive primitive curvature conditions that determine which of these two cases obtains. The paper also compares uniform reform with targeted entry expansion. A policy that targets claimants near the participation cutoff has a first-order effect on entry, while its equilibrium cutoff feedback is second order. Small budgets spent at the boundary therefore generate larger access gains than equal-budget uniform reforms. The results explain why broad administrative simplification may fail to expand access in contestable allocation systems, and why targeted support can be more effective when the objective is participation.

Keywords: Contests, Endogenous Participation, Access Reform, Public Procurement, Rent Seeking

JEL Classification: D72, D74, H23, H53

I Introduction

Many access reforms rest on a simple premise: lowering the cost of becoming an effective competitor should bring more agents into competition. Procurement reforms simplify tender documents, digitize submission systems, reduce certification burdens, and provide information about contract opportunities. Similar policies appear in grant allocation, licensing, litigation support, and public programs in which potential claimants must incur costs before they can compete. The usual prediction is inclusionary. If becoming competitively effective becomes cheaper, more agents should participate. This paper shows why that prediction can fail.

The reason is that access is often contestable rather than mechanical. A potential supplier does not merely satisfy an eligibility rule. It must acquire information, prepare documents, meet procedural requirements, and turn these inputs into a credible competitive presence. These activities are costly, and agents differ in how effectively they transform them into contest strength. A reform that lowers access costs therefore changes both the cost of producing effective competitive intensity and the competitive pressure generated by those already active. When participation is determined by an endogenous cutoff, these two effects need not point in the same direction.

I study this mechanism in a nonatomic contestable-access model with ratio-form payoffs and heterogeneous costs of producing effective competitive intensity. The leading interpretation is public procurement. Low-cost firms are already well positioned to compete; high-cost firms are closer to the participation margin. A uniform access reform helps both groups. It directly helps marginal firms by lowering their cost of becoming effective competitors, but it also strengthens low-cost incumbents. If incumbents respond strongly, aggregate competitive pressure rises and the participation cutoff moves against marginal firms. Participation can then fall even though every firm faces a lower cost.

This is the central cutoff-feedback mechanism. A uniform reform does not simply relax a participation constraint. It changes the equilibrium environment in which participation is decided. If the reform is absorbed mainly by low-cost incumbents, it raises their effective intensity and makes competition harder for marginal claimants. If the reform instead reaches the participation margin, it expands the active set. The effect of a uniform access reform therefore depends on where contest strength is produced within the active population.

The paper isolates this force by focusing on the access margin rather than the final allocation rule. In the procurement interpretation, the model does not ask which firm wins a contract after bids are submitted. It studies the prior stage in which firms decide whether, and how intensively, to become effective competitors. This is the margin affected by administrative simplification, bid-preparation assistance, certification support, and targeted outreach. The abstraction is useful because these policies often operate before the final allocation is made, but their consequences depend on the equilibrium response of all potential participants.

The first contribution is to show that equal cost reductions need not be inclusionary. In the model, a uniform access reform has a direct positive effect on marginal participation. At the same time, it increases the effective intensity of inframarginal claimants. When the inframarginal response dominates, the participation cutoff moves inward and the active set contracts. The result is not driven by a failure of implementation. The reform lowers costs for everyone. It backfires because the induced equilibrium response reallocates competitive power toward agents already best placed to compete.

The second contribution is to characterize when this reversal occurs. The relevant object is a measure of lower-tail concentration: how much effective competitive intensity is produced by the lowest-cost active claimants. If effective intensity is concentrated among these claimants, a uniform reform is absorbed by incumbents and participation contracts. If effective intensity is spread toward the participation margin, the same reform expands participation. I then show that this concentration measure can be signed from primitive curvature properties of the technology that maps costly preparation into effective contest intensity. Thus the sign of the participation response is tied to primitives, not to an assumed behavioral asymmetry between incumbents and marginal entrants.

The third contribution is to compare uniform reform with targeted entry expansion. A policy targeted exactly at the marginal type is not feasible in a continuum economy, because the marginal type has zero mass. The relevant policy treats a thin layer of claimants just outside the active set. Such a policy has a first-order effect on entry, while its equilibrium feedback through aggregate contest intensity is only second order. Small budgets spent near the participation boundary therefore generate larger access gains than equal-budget uniform reforms. The reason is simple. Boundary targeting spends resources where participation decisions are made; uniform reform spends much of the same budget on agents whose participation was never in doubt.

The procurement interpretation gives the mechanism a natural policy meaning. Public procurement systems often pursue more than allocative efficiency or low bidding costs. They also seek broad participation, competitive pressure, small-business access, and limits on incumbent dominance. The model shows why broad administrative simplification may fail on these margins. A reform that lowers costs for all firms can strengthen the firms already best positioned to compete. By contrast, support aimed at firms close to the bidding margin can expand access without requiring the policymaker to know the full effective-intensity technology, provided the policymaker can identify firms near the participation boundary.

The same logic applies beyond procurement. In many public programs, access to benefits requires costly action: information acquisition, paperwork, travel, legal compliance, or repeated interaction with administrative agencies. Reforms that lower these costs may improve average performance while generating uneven incidence across beneficiaries. For example, biometric smartcards in Andhra Pradesh improved payment efficiency and reduced leakage, but gains were not uniform across beneficiaries (Muralidharan et al., 2016). Financial-management reforms improved accountability while producing heterogeneous effects across regions and social groups (Banerjee et al., 2020). These patterns are often attributed to implementation constraints or local administrative capacity. The mechanism in this paper adds a complementary explanation: when access is contestable, broad cost reductions can shift the participation margin and concentrate gains among those already better positioned to compete.

The final part of the paper studies incidence beyond entry. A reform can change participation, contest-share concentration, and real dissipation in different directions. Uniform reform operates through the intensity profile of active claimants, so its effects depend on technology. Boundary targeting operates through the participation margin. It expands entry and dilutes incumbent contest shares, but it can raise total dissipation because new entrants exert costly effort. This distinction separates access from efficiency. Targeted entry expansion is attractive when the planner values participation or the dilution of incumbent contest power; it is not a pure efficiency result.

The broader lesson is that lowering frictions is not the same as broadening access. When participation is determined by an endogenous contest cutoff, a policy that treats all agents equally can reallocate effective competitive power toward those already best positioned to compete. Uniform access reform can therefore backfire, not because it fails to lower costs, but because it changes the equilibrium cutoff that governs participation.

A Related Literature

This paper is closest to Ritz (2008). Ritz shows that a uniform increase in contest costs can raise total effort because it weakens strong players, levels the playing field, and induces weaker players to enter. I study the opposite policy margin: a uniform reduction in access costs. The mechanism is not the reverse of Ritz's result. A cost reduction directly helps marginal claimants, but it also strengthens low-cost active claimants. When this inframarginal response is strong, aggregate contest pressure rises and the endogenous participation cutoff moves against marginal claimants. The contribution is to characterize this cutoff feedback, identify the concentration force that determines its sign, and compare uniform reform with policies targeted at the participation margin.

The paper also contributes to the literature on heterogeneity, entry, and dissipation in rent-seeking contests. Stein (2002) shows that rent dissipation can fall under mean-preserving spreads in valuations when the active set is fixed. Grandjean et al. (2017) show that this monotonicity may fail once participation responds endogenously. The mechanism here provides a cost-side analogue. A uniform reduction in marginal access costs can lower dissipation and still fail to broaden participation. More broadly, existing work studies how contestant heterogeneity and contest design shape entry, effort, and dissipation (Gradstein, 1995; Nti, 1999; Moldovanu and Sela, 2006; Fu et al., 2015; Morgan et al., 2012). I hold the contest technology fixed and study how policy-induced changes in access costs reshape the participation cutoff.

The paper is also related to work connecting conflict and lobbying outcomes to the distribution of resources and characteristics (Esteban and Ray, 1999, 2006, 2011). That literature studies how a given distribution maps into conflict intensity, lobbying, or resource allocation. I study a different question: how a policy changes who becomes an effective contestant. The relevant objects are therefore not only aggregate conflict or total effort, but also the location of the participation cutoff, the concentration of effective intensity among active claimants, and the incidence of alternative access reforms on entry, concentration, and dissipation.

Finally, the paper relates to work on policies that level the playing field within contests (Chowdhury et al., 2023). That literature studies interventions such as handicaps, affirmative action, noise, or other rule changes that alter incentives inside a contest. I instead study broad-based and boundary-targeted reductions in the cost of producing effective competitive intensity under an unchanged contest technology. This distinction is central. The main force is not a change in the contest rule holding participation fixed, but a movement of the endogenous cutoff that determines who participates.

II A Contestable-Access Model

Consider a procurement system in which participation requires more than formal eligibility. A potential supplier must learn about contract opportunities, prepare documents, satisfy administrative and legal requirements, and convert these inputs into a credible competitive bid. These activities are costly. Firms also differ in how effectively they turn preparation effort into competitive strength. Some firms can become serious competitors at low cost; others are close to the margin at which participation is no longer worthwhile.

This environment captures the central feature of contestable access. Participation is not determined only by an entry rule. It depends on the equilibrium intensity of the contest. A broad administrative reform lowers the cost of producing effective competitive intensity for all firms. This directly helps marginal suppliers because becoming competitively effective becomes cheaper. But it also helps low-cost active firms because they can now produce more competitive intensity at lower cost. If their response is strong enough, aggregate contest pressure rises and the participation cutoff moves against marginal suppliers. Participation can then fall even though the reform lowers every firm's cost.

The model isolates this cutoff-feedback mechanism. It does not describe the final procurement award rule. Instead, it studies the prior margin on which access policies operate: the costly choice of becoming an effective competitor. This is the margin affected by simplified procedures, bid-preparation support, certification assistance, information provision, and targeted outreach. The same structure also applies to grants, licenses, litigation support, and other allocation systems in which agents must incur costs before they can compete for a public resource.

The payoff $\frac{Vx}{X}$ should be interpreted as the reduced-form value of becoming competitively effective at the access stage. In procurement, x is not the final bid and $\frac{Vx}{X}$ is not a literal award rule. It captures the expected value of reaching a serious competitive position before the final allocation is made. The model therefore studies the access margin, not the final procurement auction. This interpretation is consistent with a nonatomic environment in which each potential claimant takes aggregate competitive pressure as given.

A Model and equilibrium

A primitive environment is a triple $\mathscr{E} = (V, \underline{a}, H)$. The scalar $V > 0$ is the contestable surplus. Claimants form a unit mass and are indexed by a cost type a, uniformly distributed on $[\underline{a}, 1]$, where $\underline{a} \in (0,1)$. The function $H : [0, \bar{x}) \to \mathbb{R}_+$, with $\bar{x} \in (0, \infty]$, is the common cost technology for producing effective contest intensity.

A type-a claimant who chooses intensity $x \geq 0$ pays cost $aH(x)$. The type a therefore measures how costly it is for a claimant to become competitively effective. In the procurement interpretation, low-a firms have better administrative capacity, legal knowledge, and bid-preparation infrastructure. High-a firms face greater difficulty converting preparation effort into a serious bid.

The effective-intensity formulation is related to a standard ratio-form contest through a change of variables. Suppose a finite set of claimants choose efforts e_i, produce impacts $f(e_i)$, and pay costs $ah(e_i)$. If aggregate impact is positive, claimant i's payoff is

$$\tilde{u}_i(a, e_i; e) = V \frac{f(e_i)}{\sum_j f(e_j)} - ah(e_i).$$

If f is strictly increasing and admits an inverse, define $x_i = f(e_i)$ and $H(x) = h(f^{-1}(x))$. The payoff can then be written as

$$u_i(a, x_i; x) = V \frac{x_i}{\sum_j x_j} - aH(x_i). \tag{1}$$

Thus x_i is effective contest intensity, and H is the base cost of producing that intensity. The model restricts the composite technology $H = h \circ f^{-1}$, not the effort cost h or the impact function f separately.

I impose the following assumptions on H:

$$H(0) = 0, \qquad 0 < H'(0) < \infty, \qquad H'(x) > 0, \qquad H''(x) > 0, \qquad \lim_{x \uparrow \bar{x}} H'(x) = \infty.$$

The normalization $H(0) = 0$ sets the cost of zero intensity to zero. The condition $0 < H'(0) < \infty$ gives a finite participation margin: sufficiently high-cost claimants choose zero intensity, so participation is endogenous. The inequalities $H'(x) > 0$ and $H''(x) > 0$ mean that effective intensity is costly and increasingly costly to produce. The boundary condition rules out optimal choices at the upper boundary of the intensity domain.

There is a unit mass of potential claimants. A claimant of type a chooses effective contest intensity $x \geq 0$. Let X denote aggregate effective intensity. Because claimants are nonatomic, each claimant takes X as given. If $X > 0$, a type-a claimant who chooses intensity x obtains contest payoff $\frac{Vx}{X}$ and pays cost $aH(x)$. Its payoff is $u(a, x; X) = \frac{Vx}{X} - aH(x)$.

Fix $X > 0$. Since $H'' > 0$, the objective is strictly concave in x. A type-a claimant is active if and only if the marginal payoff at zero is positive, that is, if $\frac{V}{X} > aH'(0)$. Hence participation is characterized by the cutoff $y := \frac{V}{H'(0)X}$. Types with $a < y$ choose positive intensity, while types with $a \geq y$ choose zero intensity. For an active type, the first-order condition is $\frac{V}{X} = aH'(x)$. Therefore, conditional on participation, type $a < y$ chooses $x(a; y) = (H')^{-1}\left(H'(0)\frac{y}{a}\right)$.

The cutoff is endogenous. It is defined by aggregate intensity, but aggregate intensity is produced by the types that choose to participate. An equilibrium cutoff $y > \underline{a}$ must therefore satisfy

$$\frac{1}{1-\underline{a}} \int_{\underline{a}}^{\min\{y,1\}} (H')^{-1}\left(H'(0)\frac{y}{a}\right) da = \frac{V}{H'(0)y}. \tag{2}$$

The left-hand side is the aggregate effective intensity generated by active claimants. The right-hand side is the aggregate intensity required by the definition of the cutoff.

It is useful to rewrite (2) in relative-cost units. This separates the scale of the contest from the composition of active claimants.

Definition 1 (Reduced representation) For a primitive environment $\mathscr{E} = (V, \underline{a}, H)$, define

$$g_H(t) := (H')^{-1}\left(\frac{H'(0)}{t}\right), \qquad A_H(q) := \int_q^1 g_H(t)\,dt, \qquad t, q \in (0,1].$$

For an interior cutoff $y \in (\underline{a}, 1)$, define the relative cutoff $q := \frac{\underline{a}}{y}$. Define also the scale parameter $\Theta := \frac{V(1-\underline{a})}{H'(0)\underline{a}^2}$.

The variable $t = \frac{a}{y}$ is a claimant's cost relative to the participation cutoff. The marginal claimant has $t = 1$, and therefore $g_H(1) = 0$. The function g_H gives the intensity chosen by a claimant whose cost is fraction t of the cutoff cost. Since $g_H'(t) = -\frac{H'(0)}{t^2 H''(g_H(t))} < 0$, lower relative-cost claimants choose higher intensity.

The object $A_H(q)$ aggregates these relative intensities over the active range. It captures the composition of the active set, holding fixed the scale terms collected in Θ. The parameter Θ increases with the contestable surplus V, decreases with the marginal cost $H'(0)$ of the first unit of effective intensity, and depends on the lower support point $\underline{a}$. The relative cutoff q is inversely related to participation: a lower q means a higher primitive cutoff $y = \frac{\underline{a}}{q}$, and hence a larger active set.

Using $a = yt$ and $q = \frac{\underline{a}}{y}$, the cutoff equation (2) becomes

$$A_H(q) = \Theta q^2. \tag{3}$$

This equation separates composition from scale. The left-hand side is the reduced aggregate intensity generated by active claimants. The right-hand side is the aggregate-intensity requirement implied by the contest scale and the relative cutoff.

Lemma 1 (Relative cutoff equation) *An interior cutoff $y \in (\underline{a}, 1)$ satisfies* (2) *if and only if the relative cutoff $q = \frac{\underline{a}}{y}$ satisfies*

$$A_H(q) = \Theta q^2.$$

When this condition holds, the active mass is $M(q) = \frac{\underline{a}}{1-\underline{a}}\left(\frac{1}{q} - 1\right)$.

Proposition 1 (Cutoff equilibrium) *For every primitive environment $\mathscr{E}$, the contest has a unique cutoff equilibrium, and every equilibrium has positive participation. The participation regime is determined as follows.*

(i) If $A_H(\underline{a}) > \Theta \underline{a}^2$, participation is partial. The unique relative cutoff $q \in (\underline{a}, 1)$ satisfies

$$A_H(q) = \Theta q^2.$$

(ii) If $A_H(\underline{a}) = \Theta\underline{a}^2$, the highest-cost type is exactly marginal, and the cutoff is $y = 1$.

(iii) If $A_H(\underline{a}) < \Theta\underline{a}^2$, participation is full. The unique cutoff $y > 1$ satisfies

$$\frac{1}{1-\underline{a}}\int_{\underline{a}}^{1}(H')^{-1}\left(H'(0)\frac{y}{a}\right)da = \frac{V}{H'(0)y}.$$

Proposition 1 reduces equilibrium selection to a single cutoff condition. If the cutoff is y, all types below y choose positive intensity and all types above y choose zero intensity. A higher cutoff brings in more types and raises generated aggregate intensity, while the aggregate intensity required to sustain the cutoff falls with y. These opposite monotonicities imply uniqueness.

The three regimes are determined at $y = 1$, where the highest-cost type is just indifferent. If generated intensity at this point exceeds the required intensity, the equilibrium cutoff must lie below one and participation is partial. If the two are equal, the highest-cost type is exactly marginal. If generated intensity is too low, the cutoff must lie above one and all claimants participate.

Proposition 1 gives a unique cutoff because the two sides of the consistency condition move in opposite directions. For any candidate cutoff y, active claimants generate aggregate intensity

$$\frac{1}{1-\underline{a}}\int_{\underline{a}}^{\min\{y,1\}}(H')^{-1}\left(H'(0)\frac{y}{a}\right)da.$$

This generated intensity is increasing in y: a higher cutoff admits more types and raises the intensity chosen by each already-active type. The intensity required to sustain cutoff y is $\frac{V}{H'(0)y}$, which is decreasing in y. Hence the cutoff equation has at most one solution.

The participation regime is determined by evaluating the consistency condition at $y = 1$, where the highest-cost type is just marginal. If $A_H(\underline{a}) > \Theta\underline{a}^2$, generated intensity at $y = 1$ exceeds required intensity, so the unique cutoff lies below one and participation is partial. If $A_H(\underline{a}) = \Theta\underline{a}^2$, the highest-cost type is exactly marginal. If $A_H(\underline{a}) < \Theta\underline{a}^2$, generated intensity at $y = 1$ is below required intensity, so the unique cutoff lies above one and all claimants participate.

Corollary 1 (Scale and participation) *Fix H and $\underline{a}$. Along any partial-participation equilibrium branch, participation is increasing in the scale parameter Θ. In particular,*

$$\frac{\partial q}{\partial \Theta} = \frac{q^2}{A_H'(q) - 2\Theta q} < 0, \qquad \frac{\partial M(q)}{\partial \Theta} > 0.$$

Thus a larger contestable surplus V expands participation. Conversely, if $H_\kappa = \kappa H$ for some $\kappa > 1$, participation contracts.

The scale comparative static is the benchmark. A larger Θ raises the value of participation relative to the marginal cost of the first unit of effective intensity. It lowers the relative cutoff q and expands the active mass. A proportional increase in costs has the opposite effect. If $H_\kappa = \kappa H$, then $g_{H_\kappa} = g_H$, while $\Theta_\kappa = \frac{\Theta}{\kappa}$. The relative cutoff rises, and participation falls.

The rest of the analysis focuses on partial participation, where policy can change both the size of the active set and the distribution of intensity within it.

III Uniform Access Reform

Consider a uniform cost reduction $a \mapsto a - \varepsilon$, with $0 \leq \varepsilon < \underline{a}$. The primitive type a is fixed, but its realized cost becomes $a - \varepsilon$. Let $y(\varepsilon)$ denote the post-reform cutoff in primitive type units. A primitive type a is active if and only if $a < y(\varepsilon)$.

It is useful to write the equilibrium cutoff in realized-cost units. Define

$$c(\varepsilon) := y(\varepsilon) - \varepsilon, \qquad q(\varepsilon) := \frac{\underline{a} - \varepsilon}{c(\varepsilon)}, \qquad \Theta(\varepsilon) := \frac{V(1-\underline{a})}{H'(0)(\underline{a} - \varepsilon)^2}.$$

Here $c(\varepsilon)$ is the realized participation cutoff, and $q(\varepsilon)$ is the corresponding relative cutoff. The realized cost distribution has support $[\underline{a} - \varepsilon, 1 - \varepsilon]$ and density $\frac{1}{1-\underline{a}}$. The scale parameter $\Theta(\varepsilon)$ collects the surplus, the marginal cost of the first unit of effective intensity, and the lowest realized cost.

By Lemma 1, any post-reform partial-participation equilibrium satisfies

$$A_H(q(\varepsilon)) = \Theta(\varepsilon) q(\varepsilon)^2. \tag{4}$$

The active mass is

$$M(\varepsilon) = \frac{\underline{a} - \varepsilon}{1 - \underline{a}} \left(\frac{1}{q(\varepsilon)} - 1 \right). \tag{5}$$

A uniform cost reduction changes participation through two channels. First, it lowers every realized cost, including the cost of claimants near the participation margin. This is the direct entry effect. Second, it changes the equilibrium relative cutoff $q(\varepsilon)$. This is the cutoff-feedback effect. Participation expands when the direct entry effect dominates and contracts when the cutoff feedback dominates.

The sign of this comparison is governed by how effective intensity is distributed within the active set.

Definition 2 (Lower-tail concentration) For an interior relative cutoff $q \in (\underline{a}, 1)$, define $\mathscr{C}_H(q) := \frac{(1-q) g_H(q)}{2 A_H(q)}$.

The index $\mathscr{C}_H(q)$ measures the concentration of effective intensity among the lowest-cost active claimants. Active relative costs lie in $[q,1]$. Their reduced aggregate intensity is $A_H(q)$, so average reduced intensity is $\frac{A_H(q)}{1-q}$. Hence $\mathscr{C}_H(q) = \frac{g_H(q)}{2\frac{A_H(q)}{1-q}}$. The numerator $g_H(q)$ is the intensity of the lowest-cost active claimant. The denominator is twice the average intensity among active claimants. This normalization makes the linear case the benchmark: if g_H is linear on $[q,1]$ and $g_H(1) = 0$, then $\mathscr{C}_H(q) = 1$. Values above one mean that effective intensity is concentrated near low-cost active claimants. Values below one mean that intensity is spread more evenly toward the participation margin.

Proposition 2 (Uniform reform and participation) *Suppose the initial equilibrium has partial participation, and let $q \in (\underline{a}, 1)$ solve* (3)*. Then:*

(i) There exist $\delta > 0$ and $\bar{\varepsilon} > 0$ such that, for every $\varepsilon \in [0, \bar{\varepsilon})$, equation (4) *has a unique solution $q(\varepsilon) \in (q - \delta, q + \delta)$, with $q(0) = q$.*

(ii) For all sufficiently small $\varepsilon > 0$, participation expands if $\mathscr{C}_H(q) < 1$ and contracts if $\mathscr{C}_H(q) > 1$. If $\mathscr{C}_H(q) = 1$, the first-order effect on participation is zero.

Proposition 2 is the main comparative static. A uniform cost reduction has a direct entry effect because it lowers the cost faced by marginal claimants. It also has an inframarginal intensity effect because it lowers the cost faced by active low-cost claimants. The sign of the participation response depends on which force dominates.

The concentration index $\mathscr{C}_H(q)$ measures this balance. If $\mathscr{C}_H(q) < 1$, effective intensity is sufficiently spread toward the participation margin. The direct entry effect dominates, and participation expands. If $\mathscr{C}_H(q) > 1$, effective intensity is concentrated among the lowest-cost active claimants. The reform is then absorbed mainly by inframarginal claimants, the cutoff feedback dominates, and participation contracts. Thus a uniform reform can reduce participation even though it lowers every claimant's cost.

The next result connects this equilibrium statistic to primitives. It shows when the shape of the effective-cost technology makes low-cost active claimants absorb the reform strongly enough to reduce participation.

Lemma 2 (Shape of reduced intensity) *Suppose H is three times continuously differentiable. For every $t \in (0,1)$, the sign of $g_H''(t)$ is the sign of*

$$2\left[H''(g_H(t))\right]^2 - H'(g_H(t))H'''(g_H(t)).$$

Lemma 2 connects the equilibrium concentration index to the primitive technology. The function g_H maps a claimant's cost relative to the cutoff into the intensity chosen in equilibrium. Since $g_H(1) = 0$, convexity of g_H means that intensity rises sharply as one moves from marginal claimants toward lower-cost active claimants. Concavity means that intensity is spread more evenly over the active set.

The lemma shows that this shape is determined by the curvature of the effective-cost function. The reduced-intensity schedule is locally convex when $2\left[H''(g_H(t))\right]^2 > H'(g_H(t))H'''(g_H(t))$, and locally concave when the inequality is reversed. Hence the participation effect of a uniform reform can be signed from primitives.

Proposition 3 (Primitive curvature and participation) *Suppose the initial equilibrium has partial participation, and let $q \in (\underline{a}, 1)$ solve* (3)*. Suppose also that H is three times continuously differentiable on an interval containing $[0, g_H(q)]$. Then:*

(i) If

$$2\left[H''(x)\right]^2 \geq H'(x)H'''(x) \qquad \text{for every } x \in [0, g_H(q)],$$

with strict inequality on a set of positive measure, then $M(\varepsilon) < M(0)$ for all sufficiently small $\varepsilon > 0$.

(ii) If

$$2\left[H''(x)\right]^2 \leq H'(x)H'''(x) \qquad \text{for every } x \in [0, g_H(q)],$$

with strict inequality on a set of positive measure, then $M(\varepsilon) > M(0)$ for all sufficiently small $\varepsilon > 0$.

Proposition 3 gives primitive sufficient conditions for the sign of a uniform access reform. If the curvature condition in part (i) holds, the reduced-intensity schedule is convex on the active range. Effective intensity is then concentrated among low-cost active claimants. A uniform reform is absorbed mainly by these claimants, the cutoff feedback dominates the direct entry effect, and participation falls. If the reverse condition in part (ii) holds, effective intensity is spread toward the participation margin. The direct entry effect dominates, and participation rises.

The curvature condition in Proposition 3 can be written in the primitives of the underlying ratio-form contest. Recall from (1) that $H = h \circ f^{-1}$. Let $e = f^{-1}(x)$, and define

$$\begin{aligned}\mathcal{K}_{h,f}(e) := {} & \underbrace{\left[f'(e)\right]^2 \left\{2\left[h''(e)\right]^2 - h'(e)h'''(e)\right\}}_{\text{cost curvature}} + \underbrace{\left[h'(e)\right]^2 \left\{f'(e)f'''(e) - \left[f''(e)\right]^2\right\}}_{\text{impact curvature}} \\ & - \underbrace{h'(e)h''(e)f'(e)f''(e)}_{\text{interaction}}.\end{aligned} \tag{6}$$

For $x = f(e)$,

$$2\left[H''(x)\right]^2 - H'(x)H'''(x) = \frac{\mathcal{K}_{h,f}(e)}{[f'(e)]^6}.$$

Since $f'(e) > 0$, the sign of the curvature term in Proposition 3 is the sign of $\mathcal{K}_{h,f}(e)$. Therefore, a marginal uniform cost reduction contracts participation if $\mathcal{K}_{h,f}(e) \geq 0$ on the active effort range, with strict inequality on a set of positive measure. It expands participation if $\mathcal{K}_{h,f}(e) \leq 0$ on that range, with strict inequality on a set of positive measure.

Remark 1 (Primitive curvature) Equation (6) shows that the participation effect is determined by the composite technology, not by the contest success function alone. The first term captures the curvature of effort costs. It is positive when marginal cost rises with effort but its curvature does not rise too quickly. The second term captures the curvature of the impact technology. It is positive when the proportional decline in marginal impact becomes weaker with effort. The third term is an interaction between cost curvature and impact curvature.

A familiar case pushes toward contraction. With convex effort costs and concave impact, the interaction term is positive, and the other terms may also be positive. Then $\mathcal{K}_{h,f}(e) \geq 0$, the reduced-intensity schedule is convex, and effective intensity is concentrated among low-cost active claimants. A uniform cost reduction is absorbed mainly by these claimants, so the cutoff feedback dominates and participation falls. When $\mathcal{K}_{h,f}(e) \leq 0$, intensity is less concentrated near low-cost claimants, the direct relief to marginal claimants dominates, and participation expands.

A Examples and Scope

The relevant primitive is the effective-cost function H. A ratio-form contest determines H only after both the impact technology and the primitive effort cost are specified. If the contest share is $\frac{f(e_i)}{\sum_j f(e_j)}$ and the primitive cost is $ah(e)$, effective intensity is $x = f(e)$, and $H(x) = h(f^{-1}(x))$. Thus the contest success function alone does not determine whether a uniform cost reduction expands or contracts participation. The sign depends on the composite technology $H = h \circ f^{-1}$. The examples below organize common cases by the regime they generate.

Contraction regime. A uniform reform contracts participation when the reduced-intensity schedule is convex on the active range. In this case, effective intensity is concentrated among low-cost active claimants. The reform is then absorbed mainly by inframarginal claimants, and the cutoff-feedback effect dominates the direct entry effect.

First consider a lottery contest with effective intensity $x = e$ and linear-quadratic effort cost,

$H(x) = x + \frac{\kappa x^2}{2}, \qquad \kappa > 0$. Then

$$g_H(t) = \frac{1-t}{\kappa t}, \qquad A_H(q) = \frac{q - 1 - \log q}{\kappa}.$$

The reduced-intensity schedule is convex. Equivalently, $H'(x) = 1 + \kappa x$, $H''(x) = \kappa$, and $H'''(x) = 0$, so $2\left[H''(x)\right]^2 > H'(x)H'''(x)$. By Proposition 3, a marginal uniform cost reduction contracts participation.

A second contraction example is a contest with logarithmic impact and linear effort cost. Suppose

$$\frac{\log(1+\lambda e_i)}{\sum_j \log(1+\lambda e_j)}$$

is the contest share. With $h(e) = e$, effective intensity is $x = \log(1+\lambda e)$, and $H(x) = \frac{\exp(x)-1}{\lambda}$. Hence $g_H(t) = -\log t, \qquad A_H(q) = 1 - q + q\log q$. Moreover, $H'(x) = H''(x) = H'''(x) = \frac{\exp(x)}{\lambda}$, so

$$2\left[H''(x)\right]^2 - H'(x)H'''(x) = \left(\frac{\exp(x)}{\lambda}\right)^2 > 0.$$

Thus the reduced-intensity schedule is convex, and a marginal uniform cost reduction contracts participation.

Expansion regime. A uniform reform expands participation when the reduced-intensity schedule is concave on the active range. In this case, effective intensity is spread toward the participation margin. The direct entry effect then dominates the inframarginal response.

A simple family illustrates both expansion and contraction. Suppose $H'(x) = H'(0)(1 - \lambda x)^{-p}, \qquad p > 0, \quad x < \frac{1}{\lambda}$. Then

$$g_H(t) = \frac{1 - t^{\frac{1}{p}}}{\lambda}, \qquad A_H(q) = \frac{1}{\lambda}\left[1 - q - \frac{p}{p+1}\left(1 - q^{1+\frac{1}{p}}\right)\right].$$

For this family,

$$\operatorname{sign}\left\{2\left[H''(x)\right]^2 - H'(x)H'''(x)\right\} = \operatorname{sign}(p-1).$$

Thus $p > 1$ gives the contraction regime, $p = 1$ gives first-order neutrality, and $0 < p < 1$ gives the expansion regime. The parameter p controls how sharply marginal cost rises as effective intensity approaches the technological boundary. When $p > 1$, low-cost claimants can absorb the reform strongly, and participation contracts. When $0 < p < 1$, the intensity profile is less concentrated, and participation expands.

Neutral benchmark. The case $p = 1$ in the preceding family gives the local-neutrality benchmark. Equivalently, let $H(x) = -\frac{1}{\lambda}\log(1-\lambda x)$. Then $g_H(t) = \frac{1-t}{\lambda}$, $\quad A_H(q) = \frac{(1-q)^2}{2\lambda}$. Hence $\mathscr{C}_H(q) = 1$. A marginal uniform cost reduction has no first-order effect on active mass. This benchmark is useful because it separates the contraction regime from the expansion regime.

Canonical forms outside the finite-cutoff class. Some standard contest forms do not satisfy the finite-cutoff structure used in this section. This is not a defect of the examples; it clarifies the scope of the model.

Consider the generalized Tullock contest with share $\frac{e_i^r}{\sum_j e_j^r}$ and linear effort cost. In effective-intensity units, $x = e^r$, so $H(x) = x^{\frac{1}{r}}$. If $r > 1$, then $H'(0) = \infty$. If $0 < r < 1$, then $H'(0) = 0$. If $r = 1$, then H is linear and violates strict convexity. Therefore the generalized Tullock contest with linear effort cost does not generate the finite, strictly convex participation margin studied here unless the primitive technology is modified so that the composite cost $H = h \circ f^{-1}$ satisfies $0 < H'(0) < \infty$ and $H'' > 0$.

Difference-form and logit contests also fall outside the present cutoff structure unless they are augmented with a separate entry decision. For example, if the contest share is $\frac{\exp(\sigma e_i)}{\sum_j \exp(\sigma e_j)}$, then a claimant with zero effort still obtains a positive contest share. Inactivity is therefore not governed by the marginal condition $\frac{V}{X} \leq aH'(0)$. All-pay auctions and rank-order tournaments are also outside the ratio-form formulation used here because the allocation rule is discontinuous or rank-based rather than proportional to effective intensity. They require a separate entry stage or a different equilibrium analysis.

The examples show why the composite cost H is the central primitive. Uniform access reform expands participation when the technology spreads effective intensity toward marginal claimants. It contracts participation when the technology lets low-cost active claimants absorb the reform through higher effective intensity. The standard functional form is therefore not enough; what matters is the shape of the effective-cost technology on the active range.

IV Targeted Entry Expansion

Uniform access reform is not the only way to lower the cost of becoming an effective competitor. A broad administrative reform spreads resources across all potential claimants, including agents whose participation is already secure. When low-cost active claimants absorb much of the reform, the active set can shrink. This section studies a different policy: targeting resources at claimants near the participation boundary.

In procurement, this policy corresponds to support for firms close to bidding but not yet active: bid-preparation assistance, compliance help, certification support, information provision, or targeted outreach. These policies spend resources where participation decisions are made. They therefore operate through a different margin from uniform reform. Uniform reform changes costs throughout the type distribution. Boundary targeting concentrates resources on claimants just outside the active set.

I compare the two policies at equal budget. A uniform reform $a \mapsto a - \varepsilon$ costs $B = (1-\underline{a})\varepsilon$, so a uniform reform with budget B lowers every type's cost by $\frac{B}{1-\underline{a}}$. The targeted policy spends the same budget on a thin layer of claimants near the cutoff. The question is whether concentrating resources at the participation boundary produces a larger entry response than spreading the same resources uniformly.

Let $q \in (\underline{a}, 1)$ denote the benchmark partial-participation equilibrium, and let $y(q) = \frac{\underline{a}}{q}$ be the corresponding primitive cutoff. Before policy, type a participates if and only if $a < y(q)$. The benchmark active mass is

$$M(q) = \frac{1}{1-\underline{a}} \int_{\underline{a}}^{y(q)} da = \frac{\underline{a}}{1-\underline{a}} \left(\frac{1}{q} - 1 \right).$$

Definition 3 (Equal-budget boundary policy) Fix a budget $B > 0$ and a boundary width $s > 0$, with $y(q) + s \leq 1$. The boundary layer is $\mathscr{B}_s(q) := [y(q), y(q)+s]$. The policy spends B on this layer. Each treated type receives the cost reduction $\Delta_{B,s} := \frac{B}{s}$, so realized marginal cost is $c_{B,s}(a) := a - \Delta_{B,s} \mathbf{1}\{a \in \mathscr{B}_s(q)\}$. Equivalently, $\int_{\mathscr{B}_s(q)} \Delta_{B,s} \, da = s\Delta_{B,s} = B$.

The budget is total cost reduction over the treated interval. This normalization makes boundary targeting directly comparable to the uniform benchmark, where budget B lowers every type's cost by $\frac{B}{1-\underline{a}}$. The comparison is therefore at equal total spending, not at equal treatment intensity.

The budget B measures policy size, while the width s measures how concentrated the policy is. A smaller s gives larger cost reductions to fewer types. A larger s gives smaller reductions to more types. Holding the old cutoff fixed, a treated type located $u \in [0, s]$ above $y(q)$ enters if and only if its cost reduction exceeds its distance from the cutoff, that is, if $u < \Delta_{B,s}$. The direct entry width is therefore

$$m_{B,s} := \min\{s, \Delta_{B,s}\} = \min\left\{ s, \frac{B}{s} \right\}.$$

This object measures entry at the old cutoff. It is not yet the equilibrium entry effect, because new entrants also affect aggregate contest intensity and hence the cutoff.

Let $z_{B,s}$ denote the post-policy cutoff in realized-cost units. Given $z_{B,s}$, type a is active if and only if $c_{B,s}(a) < z_{B,s}$. Assuming realized costs remain positive, the cutoff satisfies

$$\frac{1}{1-\underline{a}}\int_{\underline{a}}^{1} \mathbf{1}\{c_{B,s}(a) < z_{B,s}\} g_H\left(\frac{c_{B,s}(a)}{z_{B,s}}\right) da = \frac{V}{H'(0) z_{B,s}}. \tag{7}$$

The active mass under boundary targeting is

$$M^P(B,s) := \frac{1}{1-\underline{a}}\int_{\underline{a}}^{1} \mathbf{1}\{c_{B,s}(a) < z_{B,s}\} da.$$

For comparison, let $M^U(B)$ denote active mass under the equal-budget uniform reform. Set $b(B) := \frac{B}{1-\underline{a}}$. Then the uniform reform lowers every type's cost by $b(B)$, and

$$M^U(B) := \frac{\underline{a} - b(B)}{1-\underline{a}}\left(\frac{1}{q^U(B)} - 1\right),$$

where $q^U(B)$ solves (4) with $\varepsilon = b(B)$.

Proposition 4 (Boundary targeting and square-root entry) *Suppose the benchmark equilibrium has partial participation. Suppose also that H is twice continuously differentiable in a neighborhood of zero and* $0 < H''(0) < \infty$. *Then:*

(i) For any sequence with $B \downarrow 0$, $s \downarrow 0$, *and* $\Delta_{B,s} = \frac{B}{s} \downarrow 0$, *equation* (7) *has a unique local solution near the benchmark cutoff* $y(q)$. *Along this solution,*

$$M^P(B,s) - M(q) = \frac{m_{B,s}}{1-\underline{a}} + O\big((s+\Delta_{B,s})^2\big).$$

(ii) For small B, the direct entry margin $m_{B,s}$ *is maximized at* $s^*(B) = \sqrt{B}$, $\qquad \Delta_{B,s^*(B)} = \sqrt{B}$. *Hence*

$$M^P(B,s^*(B)) - M(q) = \frac{\sqrt{B}}{1-\underline{a}} + O(B), \qquad M^U(B) - M(q) = O(B).$$

Proposition 4 explains why boundary targeting has a larger local entry effect than an equal-budget uniform reform. A treated type enters at the old cutoff exactly when its cost reduction exceeds its distance from that cutoff. This gives the direct term $\frac{m_{B,s}}{1-\underline{a}}$. The induced cutoff response is smaller. New entrants are initially close to indifferent and therefore choose only small effective intensities. Their effect on aggregate intensity, and hence on the cutoff, is of order $(s+\Delta_{B,s})^2$.

The policy therefore faces a simple coverage-intensity tradeoff. Since $m_{B,s} = \min\left\{s, \frac{B}{s}\right\}$, a very narrow boundary layer gives large reductions to too few claimants, while a very wide layer gives reductions too small to move many claimants across the cutoff. The entry-maximizing choice balances the two margins: $s = \Delta_{B,s} = \sqrt{B}$. At this width, boundary targeting expands participation by order $\sqrt{B}$. An equal-budget uniform reform lowers every type's cost by only $\frac{B}{1-\underline{a}}$, so its local effect on participation is only of order B.

This comparison also clarifies the informational difference between the two policies. The effect of a uniform reform depends on the shape of the effective-cost technology over the active range. In particular, it depends on whether effective intensity is concentrated among low-cost incumbents or spread toward the participation margin. Boundary targeting does not require the policymaker to know the full effective-intensity technology. It does, however, require information about claimants near the participation boundary. In procurement settings, such information may come from prior bidding histories, incomplete applications, supplier registries, expressions of interest, or firms that repeatedly search for opportunities but do not submit bids.

V Extension: Incidence Beyond Entry

Participation is only one margin through which access reforms affect a contest. A reform can also change how contest power is distributed among participants and how many real resources are spent producing that power. These margins are distinct. A policy may expand the active set while increasing dissipation; it may lower dissipation while concentrating contest shares; or it may broaden participation without substantially changing aggregate intensity. This section separates these incidence effects.

Consider a partial-participation equilibrium with cutoff y and relative cutoff $q = \frac{a}{y}$. An active type $a < y$ chooses $x(a;q) = g_H\left(\frac{a}{y}\right)$, while inactive types choose zero. Average effective intensity is

$$X(q) := \frac{1}{1-\underline{a}} \int_{\underline{a}}^{y} g_H\left(\frac{a}{y}\right) da = \frac{y}{1-\underline{a}} A_H(q).$$

Define the normalized contest-share density by $\sigma(a;q) := \frac{x(a;q)}{X(q)}$, with $\sigma(a;q) = 0$ for inactive types. Hence $\frac{1}{1-\underline{a}} \int_{\underline{a}}^{1} \sigma(a;q) da = 1$.

Definition 4 (Contest-share inequality and dissipation) Contest-share inequality is the cross-sectional variance of normalized contest-share density:

$$Q(q) := \frac{1}{1-\underline{a}} \int_{\underline{a}}^{1} (\sigma(a;q) - 1)^2 \, da.$$

Dissipation is the average real cost of producing effective contest intensity:

$$D(q) := \frac{1}{1-\underline{a}} \int_{\underline{a}}^{y} aH\left(g_H\left(\frac{a}{y}\right)\right) da.$$

The object $Q(q)$ measures concentration in effective contest power. It is high when contest shares are concentrated among a small set of low-cost active claimants. Inactive claimants have zero contest shares and therefore contribute to dispersion. The object $D(q)$ measures real resource loss. The contest prize is a transfer, but producing effective intensity consumes resources. The results below are incidence results rather than welfare rankings: they show how reforms move participation, concentration, and dissipation before assigning social weights to these margins.

1 Uniform reform

A uniform reform $a \mapsto a - \varepsilon$ shifts the realized-cost distribution but leaves the effective-cost technology unchanged. The reduced-intensity schedule g_H is therefore fixed. The reform changes contest-share inequality and dissipation only through the new equilibrium cutoff and the induced reallocation of intensity across active types.

The two incidence margins are governed by different primitive objects. Contest-share inequality depends on the shape of the reduced-intensity schedule on the active range. When g_H is convex, intensity is concentrated among low-cost active claimants. A uniform reform then strengthens these claimants disproportionately and increases the concentration of contest shares. When g_H is concave, intensity is spread more evenly toward the participation margin, and the reform disperses contest shares.

Dissipation depends on where real costs are incurred along the intensity profile. The relevant object is the elasticity of the effective-cost function, $\eta_H(x) := \frac{xH'(x)}{H(x)}$. This elasticity compares marginal cost with average cost at intensity x. If η_H is increasing, high-intensity claimants operate where marginal cost is high relative to average cost. A uniform cost reduction then tends to reduce dissipation because the direct cost-saving effect dominates the induced intensity response. If η_H is decreasing, the induced response of high-intensity claimants can dominate, and dissipation may rise.

Let $Q^U(\varepsilon)$ and $D^U(\varepsilon)$ denote contest-share inequality and dissipation under the uniform reform.

Proposition 5 (Uniform reform: concentration and dissipation) *Suppose the benchmark equilibrium has partial participation, and let $q \in (\underline{a}, 1)$ solve* (3)*. Then:*

(i) If

$$2\left[H''(x)\right]^2 \geq H'(x)H'''(x) \qquad \textit{for every } x \in [0, g_H(q)],$$

then a marginal uniform reform weakly raises contest-share inequality:

$$\left.\frac{dQ^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} \geq 0.$$

If the reverse inequality holds on $[0, g_H(q)]$, then

$$\left.\frac{dQ^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} \leq 0.$$

(ii) If η_H is weakly increasing on $(0, g_H(q)]$, then a marginal uniform reform weakly lowers dissipation:

$$\left.\frac{dD^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} \leq 0.$$

If η_H is weakly decreasing on $(0, g_H(q)]$, then

$$\left.\frac{dD^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} \geq 0.$$

Proposition 5 separates two incidence margins that are often conflated. Contest-share inequality is governed by the shape of the reduced-intensity schedule. When the curvature condition in part (i) holds, g_H is convex on the active range. Effective intensity is then concentrated among low-cost active claimants, and a uniform reform increases their relative contest shares. When the reverse condition holds, intensity is spread more evenly toward the participation margin, and the reform disperses contest shares.

Dissipation is governed by a different primitive. The elasticity $\eta_H(x) = \frac{xH'(x)}{H(x)}$ determines where real resource costs are incurred along the intensity profile. If η_H is increasing, high-intensity claimants operate where marginal cost is high relative to average cost. The uniform reform then lowers dissipation because the cost-saving effect dominates the induced increase in intensity. If η_H is decreasing, the induced intensity response is strong enough to raise dissipation.

Thus a uniform reform has no fixed incidence pattern. Contest-share inequality and dissipation are governed by different primitive properties. The shape of g_H determines whether the reform concentrates or disperses contest shares, while the elasticity of H determines whether the reform raises or lowers dissipation. If g_H is convex and η_H is increasing, the reform raises contest-share inequality but lowers dissipation. This is the central tradeoff case: lower-cost active claimants absorb the reform through higher effective intensity, so contest power becomes more concentrated even though the real cost of producing intensity falls. If g_H is concave and η_H is increasing, the reform lowers both concentration and dissipation. If g_H is convex and η_H is decreasing, it raises both. If g_H is concave and η_H is decreasing, it lowers concentration but raises dissipation.

Example: bounded-impact costs. Suppose $H_p'(x) = H'(0)(1-\lambda x)^{-p}, \quad p > 0, \quad \lambda > 0, \quad x < \frac{1}{\lambda}$. This family captures environments in which effective intensity becomes increasingly costly as it approaches a technological boundary. The parameter p governs how sharply marginal cost rises near that boundary.

Normalize $H'(0) = 1$. Then

$$g_p(t) = \frac{1 - t^{\frac{1}{p}}}{\lambda}, \qquad g_p''(t) = \frac{p-1}{p^2\lambda} t^{\frac{1}{p}-2}.$$

Thus the reduced-intensity schedule is concave for $0 < p < 1$, linear for $p = 1$, and convex for $p > 1$. Moreover, η_{H_p} is increasing on the active range. The family therefore separates the share-concentration effect from the dissipation effect.

If $0 < p < 1$, a marginal uniform reform lowers both contest-share inequality and dissipation:

$$\left.\frac{dQ^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} < 0, \qquad \left.\frac{dD^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} < 0.$$

In this case, effective intensity is spread toward the participation margin. The reform disperses contest shares and lowers the real cost of producing intensity.

If $p > 1$, a marginal uniform reform lowers dissipation but raises contest-share inequality:

$$\left.\frac{dQ^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} > 0, \qquad \left.\frac{dD^U(\varepsilon)}{d\varepsilon}\right|_{\varepsilon=0} < 0.$$

This is the central tradeoff case. The reform reduces the cost of producing effective intensity, but it also lets low-cost active claimants increase intensity disproportionately. Contest power becomes more concentrated even though real dissipation falls.

2 Boundary targeting

Boundary targeting operates through the participation margin rather than the full active intensity profile. Under the balanced boundary policy, a budget B treats the layer $[y, y+\sqrt{B}]$ and gives each treated type the cost reduction $\sqrt{B}$. These are the claimants closest to entry, so the policy spends resources where participation decisions are most responsive.

The entry effect is first order. The policy moves a boundary layer of width $\sqrt{B}$ below the old cutoff, so participation expands at order $\sqrt{B}$. The induced change in aggregate intensity is smaller. New entrants are initially close to indifferent and choose only small intensities. Their aggregate contribution to effective intensity is therefore of order B. Boundary targeting consequently has a first-order effect on participation, but only a second-order effect on the cutoff, aggregate intensity, and dissipation.

Proposition 6 (Boundary targeting: local incidence) *Suppose the benchmark equilibrium has partial participation. Suppose also that H is twice continuously differentiable in a neighborhood of zero and $0 < H''(0) < \infty$. Under the balanced boundary policy, for all sufficiently small $B > 0$,*

$$M^P(B) - M(q) = \frac{\sqrt{B}}{1-\underline{a}} + O(B), \qquad Q^P(B) < Q(q), \qquad D^P(B) > D(q).$$

Moreover, if $D^P_{\text{inc}}(B)$ denotes dissipation by types that were active before the policy, then $D^P_{\text{inc}}(B) < D(q)$.

Proposition 6 shows that boundary targeting has unambiguous local incidence on entry and contest-share concentration, but not on total dissipation. The policy expands participation by moving a thin layer of previously inactive claimants into the contest. These entrants begin with small contest shares, so their entry dilutes the contest power of incumbent claimants and lowers Q. Incumbents also reduce their effective intensity, which lowers dissipation among types that were active before the policy.

Total dissipation nevertheless rises. The reason is that boundary targeting creates new costly effort at the participation margin. Although incumbent dissipation falls, the resource cost incurred by new entrants more than offsets this reduction locally. Boundary targeting therefore expands entry and reduces contest-share concentration at the cost of higher total real effort costs.

This incidence pattern differs from uniform reform. The effect of a uniform reform on concentration and dissipation depends on primitive properties of the effective-cost technology. Boundary targeting instead operates directly at the participation margin. Locally, it expands the active set, dilutes incumbent contest power, lowers incumbent dissipation, and raises total dissipation. Its appeal is therefore not that it is cost-saving. Its appeal is that it buys participation and reduces concentration. Whether that tradeoff is desirable depends on how the planner values access, concentration, and resource dissipation.

A Policy ranking with an access mandate

The incidence results do not by themselves rank policies. Boundary targeting expands participation and lowers contest-share concentration, but it raises total dissipation. A ranking therefore requires an objective that specifies how the policymaker values access, concentration, and resource costs.

I use the local reduced-form objective

$$W = \omega_M M - \omega_Q Q - \omega_D D, \qquad \omega_M, \omega_Q, \omega_D \geq 0.$$

Here ω_M is the value assigned to participation, ω_Q is the cost of concentrated contest power, and ω_D is the cost of real resource dissipation. This objective is not a welfare foundation. It is a transparent way to separate an access mandate from efficiency concerns.

The distinction matters because the square-root entry effect is not a private-surplus result. Entrants induced by a small boundary policy are close to the participation cutoff. They choose small intensities and obtain small net private returns. Boundary targeting is therefore locally attractive only if participation itself, or the dilution of incumbent contest power, has social value.

Under the balanced boundary policy,

$$M^P(B) - M(q) = \frac{\sqrt{B}}{1-\underline{a}} + O(B),$$

while the effects on concentration and dissipation are of order B. Hence, if $\omega_M > 0$,

$$W^P(B) - W(q) = \omega_M \frac{\sqrt{B}}{1-\underline{a}} + O(B) > 0$$

for all sufficiently small $B > 0$. Boundary targeting is therefore locally preferred under any objective that assigns positive direct value to access.

If $\omega_M = 0$, the square-root term disappears. Boundary targeting is then locally preferred only if

$$\omega_Q\left[Q(q) - Q^P(B)\right] > \omega_D\left[D^P(B) - D(q)\right].$$

Without an access motive, boundary targeting is not generally preferred; it must be justified by a sufficiently large reduction in contest-share concentration relative to the additional dissipation generated by new entrants.

This interpretation is natural in settings where participation is itself a policy objective. Public procurement authorities often care not only about lowering bidding costs or selecting the lowest-cost supplier, but also about broad access, competitive participation, small-business inclusion, and limits on incumbent dominance. In such settings, ω_M captures an access mandate rather than the private surplus of marginal bidders.

VI Conclusion

This paper studies access reform in contests with endogenous participation. In the model, each claimant must produce effective contest intensity before receiving any contest share. Claimants differ in the cost of producing that intensity, so a policy that lowers costs changes both individual participation incentives and the equilibrium cutoff that determines who becomes active.

A uniform access reform has two opposing effects. It lowers the cost faced by marginal claimants, which tends to expand participation. It also lowers the cost faced by low-cost active claimants, which can induce them to raise effective intensity. When this inframarginal response is strong enough, aggregate contest pressure rises and the participation cutoff moves against marginal claimants. Participation then falls even though the reform lowers every claimant's cost.

The sign of this participation response is determined by the distribution of effective intensity within the active set. A uniform reform contracts participation when effective intensity is concentrated among low-cost active claimants. It expands participation when effective intensity is spread toward the participation margin. The model links this concentration force to primitives through the curvature of the effective-cost technology. Convex reduced-intensity profiles imply contraction; concave profiles imply expansion.

Targeted entry expansion changes a different margin. Instead of reducing costs for all claimants, it spends resources on claimants just outside the participation cutoff. A boundary policy with budget B moves a layer of claimants of order $\sqrt{B}$ into participation. The induced cutoff feedback is only of order B, because new entrants initially choose small effective intensities. Boundary targeting therefore produces a larger local entry response than an equal-budget uniform reform.

The incidence results show why this comparison is not a pure efficiency ranking. Boundary targeting expands participation and reduces contest-share concentration, but it raises total dissipation because new entrants exert costly effort. It is locally attractive when the policymaker assigns direct value to participation, representation, or the dilution of incumbent contest power. Without such an access motive, boundary targeting must be justified by the reduction in concentration relative to the additional dissipation it creates.

The central implication is that equal treatment is not necessarily inclusionary when participation is determined in equilibrium. A uniform reduction in frictions can increase the effective contest power of claimants already best positioned to compete and thereby move the participation cutoff against marginal claimants. In contestable allocation systems, broad administrative simplification may therefore fail to expand access even when it lowers costs for all claimants. Targeted support can be more effective when the policy objective is to increase participation rather than only to reduce the average cost of competing.

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A Proofs for Section II

Proof of Lemma 1. Set $a = yt$ in (2). Since $da = y\,dt$, the lower limit becomes $q = \frac{\underline{a}}{y}$, and the upper limit becomes 1. Hence

$$\frac{y}{1-\underline{a}} A_H(q) = \frac{V}{H'(0)y}.$$

Using $y = \frac{\underline{a}}{q}$, this condition is equivalent to $A_H(q) = \Theta q^2$. The active mass is the measure of $[\underline{a}, y]$, divided by $1-\underline{a}$. Therefore

$$M(q) = \frac{y-\underline{a}}{1-\underline{a}} = \frac{\underline{a}}{1-\underline{a}} \left(\frac{1}{q} - 1 \right).$$

□

Proof of Proposition 1. For $y > \underline{a}$, define

$$\mathcal{F}(y) := \frac{1}{1-\underline{a}} \int_{\underline{a}}^{\min\{y,1\}} (H')^{-1} \Big(H'(0) \frac{y}{a} \Big)\, da - \frac{V}{H'(0)y}.$$

If $y \le \underline{a}$, no type is active. Aggregate intensity is then zero, so the cutoff equation cannot hold. Hence every equilibrium has positive participation.

The function $\mathcal{F}$ is strictly increasing. If $y \in (\underline{a}, 1)$, differentiation under the integral gives

$$\mathcal{F}'(y) = \frac{1}{1-\underline{a}} \int_{\underline{a}}^{y} \frac{H'(0)}{aH''\big((H')^{-1}\big(H'(0)\frac{y}{a}\big)\big)} da + \frac{V}{H'(0)y^2} > 0.$$

There is no boundary term at the upper limit because $(H')^{-1}(H'(0)) = 0$. If $y > 1$, the upper limit is fixed at 1, and the same calculation gives $\mathcal{F}'(y) > 0$. Hence the cutoff equation has at most one solution.

Existence follows from the endpoint signs. As $y \downarrow \underline{a}$, the integral term converges to zero while $\frac{V}{H'(0)y}$ remains positive, so $\lim_{y\downarrow\underline{a}} \mathscr{F}(y) < 0$. As $y \to \infty$, the integral term converges monotonically to $\bar{x}$, possibly $+\infty$, while $\frac{V}{H'(0)y} \to 0$. Since $\bar{x} > 0$, $\mathscr{F}(y) > 0$ for all sufficiently large y. Thus a unique cutoff equilibrium exists.

The regime classification follows from the sign of $\mathscr{F}(1)$. At $y = 1$, $\mathscr{F}(1) > 0$ if and only if $A_H(\underline{a}) > \Theta\underline{a}^2$. Since $\mathscr{F}$ is strictly increasing, the unique cutoff lies below, at, or above 1 according as $A_H(\underline{a})$ is greater than, equal to, or less than $\Theta\underline{a}^2$. □

Proof of Corollary 1. The interior cutoff satisfies $A_H(q) = \Theta q^2$. Let $F(q,\Theta) := A_H(q) - \Theta q^2$. Since $A'_H(q) = -g_H(q) < 0$, we have $F_q(q,\Theta) = A'_H(q) - 2\Theta q < 0$. The implicit function theorem gives

$$\frac{\partial q}{\partial \Theta} = -\frac{F_\Theta(q,\Theta)}{F_q(q,\Theta)} = \frac{q^2}{A'_H(q) - 2\Theta q} < 0.$$

Since $M(q) = \frac{\underline{a}}{1-\underline{a}}\left(\frac{1}{q} - 1\right)$, active mass is decreasing in q. Hence $\frac{\partial M(q)}{\partial \Theta} > 0$.

A larger V raises $\Theta = \frac{V(1-\underline{a})}{H'(0)\underline{a}^2}$, and therefore expands participation. If H is replaced by $H_\kappa = \kappa H$, with $\kappa > 1$, then $g_{H_\kappa} = g_H$ and $A_{H_\kappa} = A_H$, while Θ becomes $\frac{\Theta}{\kappa}$. Therefore a proportional increase in costs contracts participation. □

Proof of Proposition 2. At $\varepsilon = 0$, equation (4) reduces to $A_H(q) = \Theta q^2$. Moreover, $A'_H(q) - 2\Theta q = -g_H(q) - 2\Theta q < 0$. The implicit function theorem therefore gives $\delta > 0$, $\bar{\varepsilon} > 0$, and a unique differentiable solution $q(\varepsilon) \in (q-\delta, q+\delta)$ for every $\varepsilon \in [0,\bar{\varepsilon})$, with $q(0) = q$.

Differentiating (4) at $\varepsilon = 0$ gives

$$q'(0) = \frac{q^2\Theta'(0)}{A'_H(q) - 2\Theta q}.$$

Since $\Theta'(0) = \frac{2\Theta}{\underline{a}}$ and $A_H(q) = \Theta q^2$,

$$q'(0) = -\frac{2A_H(q)}{\underline{a}\left\{g_H(q) + \frac{2A_H(q)}{q}\right\}} < 0.$$

The active mass is $M(\varepsilon) = \frac{\underline{a}-\varepsilon}{1-\underline{a}}\left(\frac{1}{q(\varepsilon)} - 1\right)$. Differentiating at $\varepsilon = 0$ and substituting the expression for $q'(0)$ gives

$$M'(0) = \frac{2A_H(q) - (1-q)g_H(q)}{(1-\underline{a})\{qg_H(q) + 2A_H(q)\}}.$$

The denominator is positive. Therefore $M'(0) > 0$ if and only if $(1-q)g_H(q) < 2A_H(q)$, which is equivalent to $\mathscr{C}_H(q) < 1$. Similarly, $M'(0) < 0$ if and only if $\mathscr{C}_H(q) > 1$. The stated local expansion and contraction claims follow. □

Proof of Lemma 2. By definition, $H'(g_H(t)) = \frac{H'(0)}{t}$. Differentiating once gives

$$g_H'(t) = -\frac{H'(0)}{t^2 H''(g_H(t))}.$$

Differentiating again gives

$$g_H''(t) = \frac{2H'(0)}{t^3 H''(g_H(t))} - \frac{[H'(0)]^2 H'''(g_H(t))}{t^4 [H''(g_H(t))]^3}.$$

Using $H'(g_H(t)) = \frac{H'(0)}{t}$, the sign of $g_H''(t)$ is the sign of

$$2[H''(g_H(t))]^2 - H'(g_H(t))H'''(g_H(t)).$$

□

Proof of Proposition 3. By Lemma 2, the first condition implies that g_H is convex on $[q,1]$, with strict curvature on a set of positive measure. Since $g_H(1) = 0$, the graph of g_H lies below the chord joining $(q, g_H(q))$ and $(1,0)$, with strict inequality on a set of positive measure. Therefore $A_H(q) < \frac{(1-q)g_H(q)}{2}$, or equivalently $\mathscr{C}_H(q) > 1$. Proposition 2 then implies that a marginal uniform cost reduction contracts participation.

The second condition implies that g_H is concave on $[q,1]$, with strict curvature on a set of positive measure. The area inequality is then reversed: $A_H(q) > \frac{(1-q)g_H(q)}{2}$. Hence $\mathscr{C}_H(q) < 1$, and Proposition 2 implies that a marginal uniform cost reduction expands participation. □

Lemma 3 (Composite primitive curvature) *Let $H = h \circ f^{-1}$, and let $e = f^{-1}(x)$. Then $2[H''(x)]^2 - H'(x)H'''(x)$ has the same sign as*

$$\mathscr{K}_{h,f}(e) = [f'(e)]^2\{2[h''(e)]^2 - h'(e)h'''(e)\} + [h'(e)]^2\{f'(e)f'''(e) - [f''(e)]^2\} - h'(e)h''(e)f'(e)f''(e).$$

Proof. Write $A = h'(e)$, $B = h''(e)$, $C = h'''(e)$, $P = f'(e)$, $R = f''(e)$, and $T = f'''(e)$. Since $x = f(e)$,

$$H'(x) = \frac{A}{P}, \qquad H''(x) = \frac{BP - AR}{P^3}.$$

Differentiating H'' once more with respect to x gives

$$H'''(x) = \frac{CP^2 - APT - 3BPR + 3AR^2}{P^5}.$$

Therefore

$$2[H''(x)]^2 - H'(x)H'''(x) = \frac{P^2(2B^2 - AC) + A^2(PT - R^2) - ABPR}{P^6}.$$

Substituting back for A, B, C, P, R, T gives

$$2[H''(x)]^2 - H'(x)H'''(x) = \frac{\mathcal{K}_{h,f}(e)}{[f'(e)]^6}.$$

Since $f'(e) > 0$, the two expressions have the same sign. □

B Proofs for Section IV

Proof of Proposition 4. Let $y := y(q) = \frac{a}{q}$. For a boundary policy (B, s), write $\Delta_{B,s} := \frac{B}{s}$ and $c_{B,s}(a) := a - \Delta_{B,s}\mathbf{1}\{a \in [y, y+s]\}$. Define

$$\Phi_{B,s}(z) := \frac{1}{1-\underline{a}} \int_{\underline{a}}^{1} \mathbf{1}\{c_{B,s}(a) < z\} g_H\left(\frac{c_{B,s}(a)}{z}\right) da - \frac{V}{H'(0)z}.$$

The post-policy cutoff solves $\Phi_{B,s}(z) = 0$. At $B = s = 0$, the cutoff is $z = y$. Differentiating the baseline equation with respect to z gives

$$\Phi'_{0,0}(y) = \frac{1}{1-\underline{a}} \left\{ \int_{q}^{1} -t g'_H(t) dt + A_H(q) \right\}.$$

Since $g_H(1) = 0$, integration by parts gives $\int_q^1 -t g'_H(t) dt = q g_H(q) + A_H(q)$. Hence

$$\Phi'_{0,0}(y) = \frac{q g_H(q) + 2A_H(q)}{1-\underline{a}} > 0.$$

The strict derivative and the monotonicity of the cutoff equation imply a unique local solution for all sufficiently small B, s, and $\Delta_{B,s}$.

At the old cutoff y, a treated type $a = y + u$, with $u \in [0, s]$, crosses below the cutoff if and only if $u < \Delta_{B,s}$. Hence the direct boundary width is $m_{B,s} := \min\{s, \Delta_{B,s}\}$. The intensity contributed by these entrants at the old cutoff is

$$\frac{1}{1-\underline{a}}\int_0^{m_{B,s}} g_H\left(1-\frac{\Delta_{B,s}-u}{y}\right)du.$$

Since $g_H(1)=0$ and $g_H'(1)=-\frac{H'(0)}{H''(0)}$, uniformly for $u\in[0,m_{B,s}]$,

$$g_H\left(1-\frac{\Delta_{B,s}-u}{y}\right)=\frac{H'(0)}{H''(0)}\frac{\Delta_{B,s}-u}{y}+O\big((s+\Delta_{B,s})^2\big).$$

Thus entrant aggregate intensity is $O(m_{B,s}\Delta_{B,s})=O((s+\Delta_{B,s})^2)$. Since $\Phi_{0,0}'(y)>0$, the induced cutoff response satisfies $z_{B,s}-y=O((s+\Delta_{B,s})^2)$. This cutoff response changes the mass of untreated active types only at the same order. Therefore

$$M^P(B,s)-M(q)=\frac{m_{B,s}}{1-\underline{a}}+O\big((s+\Delta_{B,s})^2\big).$$

For fixed B, $m_{B,s}=\min\{s,\frac{B}{s}\}$. This expression is maximized at $s^*(B)=\sqrt{B}$, where $\Delta_{B,s^*(B)}=\sqrt{B}$. Hence

$$M^P(B,s^*(B))-M(q)=\frac{\sqrt{B}}{1-\underline{a}}+O(B).$$

For the equal-budget uniform reform, the per-type reduction is $b(B)=\frac{B}{1-\underline{a}}$. Proposition 2 gives a finite right derivative of active mass at the benchmark, so $M^U(B)-M(q)=O(B)$. □

C Derivations for contest-share inequality and dissipation

At an interior equilibrium, $y=\frac{a}{q}$ and $x(a)=g_H(\frac{a}{y})$ for $a<y$. Average effective intensity is

$$X(q)=\frac{1}{1-\underline{a}}\int_{\underline{a}}^{y} g_H\left(\frac{a}{y}\right)da=\frac{yA_H(q)}{1-\underline{a}},\qquad A_H(q):=\int_q^1 g_H(t)dt.$$

Define $S_H(q):=\int_q^1 g_H(t)^2dt$ and $\Psi_H(q):=\int_q^1 tH(g_H(t))dt$.

Lemma 4 (Reduced forms) *At an interior equilibrium,*

$$Q(q)=\frac{(1-\underline{a})qS_H(q)}{\underline{a}A_H(q)^2}-1,\qquad D(q)=\frac{\underline{a}^2\Psi_H(q)}{q^2(1-\underline{a})}.$$

Using the cutoff equation,

$$D(q)=\frac{V}{H'(0)}\frac{\Psi_H(q)}{A_H(q)}.$$

Proof. Since σ has mean one, $Q(q) = \frac{1}{1-\underline{a}} \int_{\underline{a}}^{1} \sigma(a;q)^2 da - 1$. Only active types have positive intensity. Substituting $t = \frac{a}{y}$ gives

$$\frac{1}{1-\underline{a}} \int_{\underline{a}}^{y} \frac{g_H\left(\frac{a}{y}\right)^2}{X(q)^2} da = \frac{yS_H(q)}{(1-\underline{a})X(q)^2}.$$

Using $X(q) = \frac{yA_H(q)}{1-\underline{a}}$ and $y = \frac{\underline{a}}{q}$ yields the expression for $Q(q)$.

The expression for $D(q)$ follows from the same substitution:

$$D(q) = \frac{1}{1-\underline{a}} \int_{\underline{a}}^{y} aH\left(g_H\left(\frac{a}{y}\right)\right) da = \frac{y^2}{1-\underline{a}} \int_{q}^{1} tH(g_H(t))dt.$$

Since $y = \frac{\underline{a}}{q}$, this gives $D(q) = \frac{\underline{a}^2 \Psi_H(q)}{q^2(1-\underline{a})}$. The cutoff equation implies $\frac{\underline{a}^2}{q^2(1-\underline{a})} = \frac{V}{H'(0)A_H(q)}$. Substitution gives the final expression. □

Lemma 5 (Moment comparison) *Let $g : [q,1] \to \mathbb{R}_+$ be continuous, nonincreasing, and satisfy $g(1) = 0$. Let $A = \int_q^1 g(t)dt$ and $S = \int_q^1 g(t)^2 dt$. If g is convex, then $2Ag(q) \geq 3S$. If g is concave, then $2Ag(q) \leq 3S$. The inequality is strict when the corresponding curvature is strict on a set of positive measure.*

Proof. Normalize the interval to $[0,1]$ and divide by $g(q) > 0$. The claim becomes $\int_0^1 h^2 \leq \frac{2}{3} \int_0^1 h$ for convex h, and the reverse inequality for concave h, where $h(0) = 1$, $h(1) = 0$, and $h \geq 0$. For convex h, the layer-cake representation gives $\int h = \int_0^1 R(s)ds$ and $\int h^2 = \int_0^1 2sR(s)ds$, where $R(s) = |\{t : h(t) \geq s\}|$. Convexity of h implies that R is convex and decreasing, with $R(1) = 0$. A convex decreasing function with endpoint $R(1) = 0$ is a nonnegative mixture of hinge functions $(r-s)_+$. For each hinge function,

$$\int_0^1 (1-3s)(r-s)_+ ds = \frac{r^2(1-r)}{2} \geq 0.$$

Integrating over the mixture gives $\int_0^1 (1-3s)R(s)ds \geq 0$, which is equivalent to $\int h^2 \leq \frac{2}{3} \int h$. The concave case is identical with the inequalities reversed. Strict curvature on a set of positive measure gives a strict inequality. □

Lemma 6 (Uniform reform derivatives) *Let $Q^U(\varepsilon)$ and $D^U(\varepsilon)$ denote contest-share inequality and dissipation under the uniform reform $a \mapsto a - \varepsilon$. At $\varepsilon = 0$,*

$$\frac{d}{d\varepsilon} \log\left(Q^U(\varepsilon) + 1\right) = \frac{qg_H(q)}{\underline{a}\{2A_H(q) + qg_H(q)\}} \left[\frac{2A_H(q)g_H(q)}{S_H(q)} - 3\right],$$

and

$$\frac{d}{d\varepsilon}\log D^U(\varepsilon) = \frac{2q}{\underline{a}\{2A_H(q)+qg_H(q)\}}\left[\frac{A_H(q)qH(g_H(q))}{\Psi_H(q)} - g_H(q)\right].$$

Proof. Let $\alpha(\varepsilon) = \underline{a} - \varepsilon$. The post-reform cutoff $q(\varepsilon)$ solves

$$A_H(q(\varepsilon)) = \frac{V(1-\underline{a})}{H'(0)\alpha(\varepsilon)^2}q(\varepsilon)^2.$$

Differentiating at zero gives

$$q'(0) = -\frac{2A_H(q)q}{\underline{a}\{2A_H(q)+qg_H(q)\}}.$$

The uniform reform gives

$$Q^U(\varepsilon)+1 = \frac{(1-\underline{a})q(\varepsilon)S_H(q(\varepsilon))}{\alpha(\varepsilon)A_H(q(\varepsilon))^2}, \qquad D^U(\varepsilon) = \frac{\alpha(\varepsilon)^2\Psi_H(q(\varepsilon))}{q(\varepsilon)^2(1-\underline{a})}.$$

Taking logarithms, differentiating at $\varepsilon = 0$, and using $A'_H(q) = -g_H(q)$, $S'_H(q) = -g_H(q)^2$, and $\Psi'_H(q) = -qH(g_H(q))$ gives the two displayed formulas. □

Proof of Proposition 5. For part (i), Lemma 2 implies that the displayed curvature condition makes g_H convex on the active range. Lemma 5 then gives $2A_H(q)g_H(q) \geq 3S_H(q)$. Lemma 6 therefore implies $Q^{U\prime}(0) \geq 0$. If the reverse curvature condition holds, then g_H is concave on the active range, the moment inequality is reversed, and $Q^{U\prime}(0) \leq 0$.

For part (ii), $g_H(t) = x$ is equivalent to $H'(x) = \frac{H'(0)}{t}$. Hence

$$\frac{tH(g_H(t))}{g_H(t)} = \frac{H'(0)}{\eta_H(g_H(t))}.$$

If η_H is increasing, then this ratio is increasing in t, because g_H is decreasing. Therefore its value at q is weakly below its average with weights $g_H(t)$:

$$\frac{qH(g_H(q))}{g_H(q)} \leq \frac{\int_q^1 tH(g_H(t))dt}{\int_q^1 g_H(t)dt} = \frac{\Psi_H(q)}{A_H(q)}.$$

Lemma 6 then gives $D^{U\prime}(0) \leq 0$. If η_H is decreasing, the weighted-average inequality is reversed, and $D^{U\prime}(0) \geq 0$. □

Lemma 7 (Balanced boundary expansions) *Let $B = r^2$, and consider the balanced boundary policy that treats $[y, y+r]$ and gives each treated type the cost reduction r. Let z_B be the* *post-policy cutoff. Define $A = A_H(q)$, $S = S_H(q)$, $g = g_H(q)$, and $R = 2A + qg$. Then, as $B \downarrow 0$,*

$$z_B - y = -\frac{\frac{H'(0)}{H''(0)}}{2yR}B + O(B^{3/2}).$$

Moreover,

$$Q^P(B) - Q(q) = -\Lambda_Q(q)B + O(B^{3/2}),$$

where

$$\Lambda_Q(q) = \big(Q(q) + 1\big)\frac{\frac{H'(0)}{H''(0)}}{2y^2R}\left[3 + \frac{qg^2}{S}\right] > 0.$$

Finally,

$$D^P_{\text{inc}}(B) - D(q) = -\frac{[H'(0)]^2}{H''(0)}\frac{A + qg}{2(1-\underline{a})R}B + O(B^{3/2}),$$

and

$$D^P(B) - D(q) = \frac{[H'(0)]^2}{H''(0)}\frac{A}{2(1-\underline{a})R}B + O(B^{3/2}).$$

Proof. Let $k := \frac{H'(0)}{H''(0)}$. A treated type $a = y + u$, with $u \in [0, r]$, has realized cost $y + u - r$. At the old cutoff y, its relative realized cost is $1 - \frac{r-u}{y}$. Since $g_H(1) = 0$ and $g'_H(1) = -k$,

$$g_H\left(1 - \frac{r-u}{y}\right) = k\frac{r-u}{y} + O(B).$$

Hence the entrant contribution to unnormalized aggregate intensity is

$$\int_0^r g_H\left(1 - \frac{r-u}{y}\right)du = \frac{kB}{2y} + O(B^{3/2}).$$

The derivative of the baseline cutoff residual, in unnormalized units, is $R = 2A + qg$. Therefore the post-policy cutoff satisfies

$$z_B - y = -\frac{kB}{2yR} + O(B^{3/2}).$$

Let I_B and J_B denote the unnormalized first and second moments of effective intensity after the policy. The cutoff equation gives $I_B = \frac{(1-\underline{a})V}{H'(0)z_B}$, so $I_B - yA = -A(z_B - y) + O(B^{3/2})$. Entrant second moments are $O(B^{3/2})$. For incumbents, $J(z) = zS_H(\frac{\underline{a}}{z})$, so $J'(y) = S + qg^2$. Hence

$J_B - yS = (S + qg^2)(z_B - y) + O(B^{3/2})$. Since $Q + 1 = (1 - \underline{a})\frac{J}{I^2}$,

$$Q^P(B) - Q(q) = (Q(q) + 1)\left[\frac{S + qg^2}{yS} + \frac{2}{y}\right](z_B - y) + O(B^{3/2}).$$

Substituting the expression for $z_B - y$ gives the displayed formula for $Q^P(B) - Q(q)$.

It remains to compute dissipation. Let $K(z)$ be incumbent unnormalized dissipation at cutoff z. Then

$$K(z) = z^2 \Psi_H\left(\frac{a}{z}\right).$$

Using $\Psi_H'(q) = -qH(g_H(q))$, we obtain

$$K'(y) = y\{2\Psi_H(q) + q^2 H(g)\}.$$

The identity $2\Psi_H(q) + q^2 H(g) = H'(0)\{A + qg\}$ follows by integrating the derivative of $t^2 H(g_H(t))$ over $[q, 1]$. Therefore

$$K'(y) = yH'(0)(A + qg).$$

Substituting $z_B - y = -\frac{kB}{2yR} + O(B^{3/2})$ gives the incumbent dissipation expansion after division by $1 - \underline{a}$.

Entrants add unnormalized dissipation

$$\int_0^r (y + u - r) H\left(g_H\left(1 - \frac{r - u}{y}\right)\right) du = \frac{[H'(0)]^2}{2H''(0)} B + O(B^{3/2}).$$

Adding this entrant term to the incumbent expansion gives the total dissipation expansion. □

Proof of Proposition 6. By Proposition 4, the balanced boundary policy satisfies

$$M^P(B) - M(q) = \frac{\sqrt{B}}{1 - \underline{a}} + O(B).$$

Lemma 7 gives $Q^P(B) - Q(q) = -\Lambda_Q(q)B + O(B^{3/2})$, with $\Lambda_Q(q) > 0$. Hence $Q^P(B) < Q(q)$ for all sufficiently small $B > 0$. The same lemma gives $D^P_{\text{inc}}(B) - D(q) < 0$ and $D^P(B) - D(q) > 0$ for all sufficiently small $B > 0$. These inequalities prove the proposition. □

Lemma 8 (Bounded-impact family) *Suppose $H_p'(x) = H'(0)(1 - \lambda x)^{-p}$, with $p > 0$, $\lambda > 0$, and*

$x < \frac{1}{\lambda}$. *After normalizing* $H'(0) = 1$,

$$g_p(t) = \frac{1 - t^{\frac{1}{p}}}{\lambda}, \qquad g_p''(t) = \frac{p-1}{p^2\lambda} t^{\frac{1}{p}-2}.$$

Thus g_p *is concave for* $0 < p < 1$, *linear for* $p = 1$, *and convex for* $p > 1$. *Moreover,* η_{H_p} *is increasing on* $(0, \frac{1}{\lambda})$.

Proof. The expression for g_p follows from $H_p'(g_p(t)) = \frac{1}{t}$. Differentiating $g_p(t) = \frac{1-t^{1/p}}{\lambda}$ twice gives the displayed expression for g_p'', which signs the curvature of g_p.

It remains to show that η_{H_p} is increasing. Let $z = \lambda x \in (0,1)$. For $p \neq 1$,

$$H_p(x) = \frac{(1-z)^{1-p} - 1}{\lambda(p-1)}, \qquad \eta_{H_p}(x) = \frac{(p-1)z(1-z)^{-p}}{(1-z)^{1-p} - 1}.$$

A direct derivative with respect to z gives

$$\frac{d\eta_{H_p}}{dz} = \frac{(p-1)(1-z)^{-p-1}\left\{1 - (1-z)^p - pz(1-z)^{p-1}\right\}}{\left((1-z)^{1-p} - 1\right)^2}.$$

The numerator has the same sign as $p-1$, because $1-(1-z)^p - pz(1-z)^{p-1}$ is positive for $p > 1$ and negative for $0 < p < 1$. Hence $\frac{d\eta_{H_p}}{dz} > 0$ for all $p \neq 1$. For $p = 1$, $H_p(x) = -\frac{1}{\lambda}\log(1-\lambda x)$, and

$$\eta_{H_p}(x) = \frac{z}{(1-z)[-\log(1-z)]},$$

which is increasing on $(0,1)$. Since $z = \lambda x$, η_{H_p} is increasing in x. □