Timing and Targeting in Network Games*

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Abstract

We study how the timing of intervention affects optimal targeting in network games with strategic complementarities and incomplete information. Individuals differ in their number of connections and choose effort before knowing who their neighbors will be. A social planner allocates a fixed enforcement budget across degree classes to minimize total surviving activity. When enforcement and effort are chosen simultaneously, the unique equilibrium targets the most connected individuals and equalizes effort across targeted classes, without fully eliminating any class. When the planner can commit to enforcement before individuals choose effort, however, it anticipates how intervention reduces effort not only among those directly targeted but also throughout the network. We show that these indirect effects, captured by a measure of network amplification, can make it optimal to concentrate enforcement more heavily on highly connected individuals. In particular, when the importance of a degree class in individuals' beliefs about their neighbors, relative to its population size, weakly increases with degree, network amplification is stronger for more connected individuals. Under this condition, whenever simultaneous intervention targets at least two degree classes, reallocating some enforcement toward the more connected class strictly reduces total surviving activity. Finally, we identify an exact enforcement-budget threshold above which the ability to commit strictly improves outcomes relative to simultaneous intervention.

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1 Introduction

Interventions in social networks affect not only those directly targeted but also the behavior of others through network interactions. Whether disrupting a criminal organization, limiting the spread of misinformation, or containing an epidemic, a social planner must account for how individuals respond to intervention. In particular, individuals choose their effort based on both the likelihood that their activity survives intervention and the effort they expect from their neighbors. Through strategic complementarities, changes in individual effort can propagate throughout the network, amplifying the effects of intervention. This raises a fundamental question: how does the timing of intervention affect optimal targeting when individuals only know their direct connections? To address this question, we compare two settings in which the planner and individuals choose their actions either simultaneously or sequentially.

When intervention and effort are chosen simultaneously, the planner takes individuals' effort as given when allocating enforcement. By contrast, when decisions are sequential, the planner commits to an enforcement policy before individuals choose their effort and therefore anticipates how they will respond, both directly and through network interactions. As a result, a targeting policy that is optimal under simultaneous decisions need not remain optimal when the planner can commit. This distinction is closely related to the classic trade-off between commitment and discretion highlighted by Kydland and Prescott (1977). Our analysis examines how this trade-off shapes the allocation of enforcement across individuals with different levels of network connectivity.

We consider an environment in which the planner observes degree types but not individual links and players know their own degree types but not the identities of their future neighbors. We first show that the simultaneous game has a unique type-symmetric equilibrium in which the planner targets an initial block of the highest-degree classes, never fully removes a class, and equalizes effort across all targeted classes. Taking effort as given, the planner allocates enforcement to the classes with the highest effort. If two partially targeted classes choose different levels of effort, shifting enforcement toward the higher-effort class would lower total surviving activity. Full removal of a class cannot occur in equilibrium because that class would exert no effort, giving the planner an incentive to redirect enforcement toward a class with positive effort.

The equalization of effort in the simultaneous equilibrium provides the key to understanding how commitment changes optimal targeting. When the planner commits to an enforcement policy before players choose effort, it anticipates how intervention affects not only the effort of those directly targeted but also the effort of others through network complementarities. The resulting marginal effect of enforcement has two components: a direct effect, proportional to individual effort, and an indirect effect, capturing the amplification of behavioral responses throughout the network. While the direct effect is equalized across targeted classes in the simultaneous equilibrium, the amplification effect may differ across classes. To capture these differences, we introduce a class-specific *amplification score*. We show that whenever two targeted classes have different amplification scores, shifting a small amount of enforcement toward the class with the higher score reduces total surviving activity once individuals adjust their effort. We refer to this discrepancy between the equalization of effort under simultaneous decisions and the full marginal effects of enforcement under commitment as the *equalization distortion*.

A restriction on neighbor beliefs allows us to rank amplification scores by degree. Suppose that higher-degree classes receive weakly greater neighbor-belief weight per unit of population mass. This restriction is satisfied by link-based sampling and is motivated by the friendship

paradox (Feld, 1991; Jackson, 2019; Airoldi and Christakis, 2024; Charness et al., 2026). Under this restriction, amplification scores strictly increase with degree. Hence, whenever the simultaneous policy targets at least two classes, shifting a sufficiently small amount of enforcement within the targeted block from a lower-degree class to a higher-degree class reduces the sequential objective. Importantly, this result concerns only enforcement intensity within the block, since the planner gives priority to higher-degree classes under both timings.

Under the same restriction on neighbor beliefs, the baseline model identifies the exact enforcement budget threshold above which commitment strictly reduces total surviving activity relative to the simultaneous policy. At or below the threshold, the simultaneous and sequential planners each allocate their entire budget to the highest-degree class. Above the threshold, the simultaneous planner targets at least two degree classes.

Furthermore, the gain from commitment reaches its maximum at an enforcement budget strictly between this threshold and the budget required to set every class's survival probability to zero. As the enforcement budget approaches this upper limit, network spillovers vanish and the gain from commitment tends to zero. For a sufficiently small fixed budget above the initial threshold, stronger complementarities can make the highest-degree class attractive enough that the simultaneous planner spends its entire budget on that class. Both planners then choose the same policy, so the gain from commitment disappears.

We also characterize the structure of optimal sequential targeting under the same belief restriction. We show that the highest-degree classes may be fully removed, an intermediate block may be partially targeted, and the lowest-degree classes may be left untargeted, with any of these regions possibly empty. Within the partially targeted block, survival increases as degree falls, and the planner equalizes the full marginal cost of survival. Hence, commitment changes the margin that is equalized in equilibrium and can make full removal of a class optimal, whereas every class has a positive survival probability in the simultaneous equilibrium.

Finally, we study when simultaneous intervention targets more than one class. A larger enforcement budget lowers equilibrium spillovers and effort but weakly expands targeting toward lower-degree classes. Stronger complementarities raise spillovers and effort while weakly contracting the targeted block; a shift of neighbor beliefs toward higher-degree types weakly raises spillovers and effort and weakly contracts the block. These comparisons identify when the within-block distortion can arise; they do not, by themselves, rank its magnitude. A numerical example illustrates the possible size of the difference: total surviving activity under the sequential benchmark can be more than ten percent below its simultaneous-equilibrium value while the stability condition is satisfied. The amplification score identifies the direction of local improvement using the degree distribution, neighbor beliefs, and equilibrium spillover index that also enter the simultaneous allocation.

1.1 Related literature

Closest to our targeting problem is the literature on intervention in network games. In a fixed network, Ballester et al. (2006) characterize the "key player" whose removal most reduces aggregate activity, and Lee et al. (2021) apply this approach to juvenile-delinquency networks. Galeotti et al. (2020) study optimal intervention with strategic spillovers,[1] while Li et al. (2022) analyze limits to targeting. Chaudhuri (2026) shows that the key player can change when the removal is unanticipated and observed only by the removed player's neighbors. The effect of a

[1] See also Kor et al. (2025).

removal then depends on the observers' beliefs about which of their neighbors also observed it. In our model, by contrast, players anticipate enforcement, so the timing of the planner's choice matters: we compare the enforcement allocation when the planner moves simultaneously with players and when it commits before they choose effort.

The informational environment builds on Galeotti et al. (2010), who study network games in which players know their own type but not their realized local network, and de Martí and Zenou (2015), who establish existence and uniqueness under strategic complementarities and incomplete information. Chaudhuri et al. (2026) study how beliefs about the network affect behavior under peer effects. In our model, targeting is itself a choice of the planner. Players' anticipated survival probabilities affect effort and, through that effort, the activity they expect from their neighbors.

The timing comparison also relates to the distinction between commitment and discretionary policy (Kydland and Prescott, 1977). Bester and Strausz (2001) and Doval and Skreta (2022) study limited commitment in mechanism design. We focus on a particular source of the gain from commitment: effort equalization leaves unequal network amplification across targeted classes. The amplification score isolates this margin and identifies a locally improving reallocation without requiring a solution of the full sequential problem.

Finally, the restriction on neighbor beliefs is motivated by the friendship paradox (Feld, 1991; Jackson, 2019): a randomly sampled neighbor has higher expected degree than a randomly sampled individual. Charness et al. (2026) provide causal evidence on the behavioral consequences of this bias. We impose a class-level ordering of neighbor-belief weight relative to population mass, which link-based sampling satisfies. This ordering makes amplification scores strictly increase with degree, so commitment strictly reduces total surviving activity whenever the simultaneous planner targets at least two classes.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the simultaneous equilibrium. Section 4 compares it with the sequential benchmark. Section 5 studies comparative statics. The Appendix contains proofs, the global degree-ordering result for sequential targeting, and robustness analyses.

2 Model

Consider a social planner and a continuum of players of total mass n. Players differ in their observable degree type, and their efforts are strategic complements. Let $K = \{k_1 > k_2 > \cdots > k_d > 0\}$ denote the set of degree types, ordered from highest to lowest, and let $m_\ell > 0$ be the mass of type-ℓ players, with $\sum_{\ell=1}^{d} m_\ell = n$.

A type-ℓ player has k_ℓ neighbors when the network is realized. The planner observes degree types but not individual links. As in Galeotti et al. (2010), players choose effort before the network is realized. Each player knows her own degree type but not the identities or degree types of her future neighbors.

We assume that all players hold the same beliefs about a future neighbor's degree, independently of their own type.

Assumption 1 (Type-independent neighbor beliefs) Players believe that their future neighbors' degree types are drawn independently from a common distribution $\pi = (\pi_1, \ldots, \pi_d)$, with $\pi_j > 0$ for all j and $\sum_{j=1}^{d} \pi_j = 1$. The distribution is independent of the player's own degree type and of the realized network.

Under Assumption 1, players of the same degree type are ex ante identical. The distribution π describes their beliefs about a future neighbor and need not equal the population distribution of degree types. We leave the origin of these beliefs unrestricted.

Policy and budget. The planner observes degree types but not realized identities or links. It chooses a type-based targeting rule $\mathbf{p} = (p_1, \ldots, p_d)$, where $p_\ell \in [0,1]$ is the probability that the activity of a type-ℓ player is removed before payoffs are realized. Let $q_\ell := 1 - p_\ell$ denote the corresponding survival probability, and write $\mathbf{q} = (q_1, \ldots, q_d) \in [0,1]^d$.

The enforcement budget satisfies $\sum_{\ell=1}^{d} m_\ell p_\ell = B$, where $B \in (0, n)$. Equivalently, the feasible set of survival profiles is[2]

$$\mathcal{Q}(B) := \left\{ \mathbf{q} \in [0,1]^d : \sum_{\ell=1}^{d} m_\ell (1 - q_\ell) = B \right\}.$$

The two games differ in whether the planner can commit to a survival profile before players choose effort. In the simultaneous game, players know the planner's objective, budget, and information but choose effort using a conjectured survival profile $\mathbf{q}^e$. The planner chooses a survival profile $\mathbf{q}$ at the same time, taking effort as given. Equilibrium requires the conjecture to be correct, so that $\mathbf{q}^e = \mathbf{q}$.

In the sequential game, the planner first commits to the targeting rule and its associated survival profile $\mathbf{q}$. Players observe this profile before choosing effort.[3] Preferences and information about realized links are the same under both timings.

Since players with the same degree type are ex ante identical, we restrict attention to type-symmetric effort profiles. Let $x_\ell \geq 0$ denote the effort of a type-ℓ player, and write $\mathbf{x} = (x_1, \ldots, x_d)$. Expected neighbor effort is $\bar{x} := \sum_{\ell=1}^{d} \pi_\ell x_\ell$. Given $(\mathbf{x}, \mathbf{q})$, a type-ℓ player has expected payoff

$$U_\ell(\mathbf{x}, \mathbf{q}) = \underbrace{\alpha q_\ell x_\ell}_{\text{private return}} - \underbrace{\frac{1}{2} x_\ell^2}_{\text{effort cost}} + \underbrace{\phi k_\ell q_\ell x_\ell \bar{x}}_{\text{network complementarity}}, \tag{1}$$

where $\alpha > 0$ is the marginal private return to effort and $\phi > 0$ measures the strength of strategic complementarities. The private and network returns are obtained only if the player's activity survives, whereas the effort cost is incurred regardless. A player with more neighbors obtains a larger network return from a given level of expected neighbor effort.

Remark 1 (Timing and the spillover aggregate) The simultaneous benchmark isolates a pre-enforcement investment channel. Targeting a degree class lowers its incentive to exert effort, which changes expected neighbor effort through equilibrium behavior. A planner that cannot commit before effort is chosen does not internalize this induced response when allocating enforcement. Another natural specification allows complementarities to operate through post-enforcement interactions with surviving neighbors. The relevant spillover aggregate is then expected surviving-neighbor effort, $\sum_{j=1}^{d} \pi_j q_j x_j$. Section 4.7 and Appendix B.1 study this case. The equalization logic remains, but the amplification margin changes because enforcement now

[2] Restricting attention to class-level survival probabilities is without loss of generality for anonymous enforcement policies. Since the planner observes degree type but not identity, realized links, or effort, players within a class are observationally identical. Any feasible anonymous policy is therefore outcome-equivalent to some $\mathbf{q} \in [0,1]^d$.

[3] Notice that in both games, players choose effort before learning whether their activity survives.

affects both a player's return to effort and the neighbor activity that enters the complementarity term.

Define the planner's fixed-effort objective by

$$W(\mathbf{x}, \mathbf{q}) := \sum_{\ell=1}^{d} m_\ell q_\ell x_\ell,$$

which measures total surviving activity under effort profile $\mathbf{x}$ and survival profile $\mathbf{q}$.[4]

Definition 1 (Type-symmetric simultaneous equilibrium) A type-symmetric pure-strategy simultaneous equilibrium is a triple $(\mathbf{q}^e, \mathbf{q}^*, \mathbf{x}^*)$ such that, for each degree type ℓ, x_ℓ^* solves

$$\max_{x \geq 0} \left\{ \alpha q_\ell^e x - \frac{1}{2}x^2 + \phi k_\ell q_\ell^e x \bar{x}^* \right\}, \qquad \bar{x}^* := \sum_{j=1}^{d} \pi_j x_j^*,$$

the planner chooses

$$\mathbf{q}^* \in \arg\min_{\mathbf{q} \in \mathcal{Q}(B)} W(\mathbf{x}^*, \mathbf{q}),$$

and the players' conjecture about the survival profile matches the planner's choice, so that $\mathbf{q}^e = \mathbf{q}^*$. Since equilibrium requires $\mathbf{q}^e = \mathbf{q}^*$, we henceforth suppress $\mathbf{q}^e$ and denote a simultaneous equilibrium by $(\mathbf{q}^*, \mathbf{x}^*)$.

Because the population is a continuum, a single player's deviation does not affect aggregate expected neighbor effort, $\bar{x}^*$. Each player therefore takes $\bar{x}^*$ as given when choosing effort.

Let $\bar{k} := \sum_{\ell=1}^{d} \pi_\ell k_\ell$ denote the belief-weighted mean degree. We impose the following stability condition.

Assumption 2 (Network stability) The network complementarity parameter $\phi > 0$ satisfies $\phi \bar{k} < 1$.

Assumption 2 bounds the strength of complementarities, as is standard in network games (Ballester et al., 2006). It ensures that the spillover term below is well defined for every feasible survival profile.

3 The simultaneous equilibrium

This section characterizes the simultaneous equilibrium. We first derive effort for a given survival profile and then use the planner's incentives to establish cutoff targeting and effort equalization.

[4]The planner's objective can be interpreted as a reduced-form social loss from surviving activity. More generally, the planner may maximize $-H(W(\mathbf{x}, \mathbf{q}))$, where H is strictly increasing. The baseline takes H to be linear. A strictly increasing H preserves the planner's policy ranking. Appendix B.3 extends the local equalization-distortion result to separable convex social costs.

Definition 2 (Spillover index) For any feasible survival profile $\mathbf{q} = (q_1, \ldots, q_d)$, define

$$\mu(\mathbf{q}) := \sum_{\ell=1}^{d} \pi_\ell q_\ell, \qquad \kappa(\mathbf{q}) := \sum_{\ell=1}^{d} \pi_\ell k_\ell q_\ell.$$

The spillover index associated with $\mathbf{q}$ is given by

$$L(\mathbf{q}) := \frac{\phi\, \mu(\mathbf{q})}{1 - \phi\, \kappa(\mathbf{q})}.$$

Assumption 2 imposes $1 - \phi\kappa(\mathbf{q}) > 0$ for every feasible survival profile because $\kappa(\mathbf{q}) \leq \bar{k}$. Thus $L(\mathbf{q})$ is well defined. Since $B < n$, some class has $q_\ell > 0$. With $m_\ell > 0$ and $\pi_\ell > 0$ for every ℓ, we have $\mu(\mathbf{q}) > 0$, and hence $L(\mathbf{q}) > 0$.

The index combines the direct effect of complementarities, $\phi\, \mu(\mathbf{q})$, with the social multiplier $\frac{1}{1-\phi\, \kappa(\mathbf{q})} > 1$, familiar from network games (Glaeser et al., 2003; Ballester et al., 2006; Zenou, 2025). Higher effort raises expected neighbor effort, which induces further effort. The denominator summarizes this feedback.

Fix a feasible survival profile $\mathbf{q}$. Given expected neighbor effort $\bar{x}$, the unique best response of a type-ℓ player is $x_\ell = \alpha q_\ell + \phi k_\ell q_\ell \bar{x}$. In a type-symmetric equilibrium, expected neighbor effort must satisfy $\bar{x} = \sum_{\ell=1}^{d} \pi_\ell x_\ell$. Substituting the best responses into this consistency condition yields $\bar{x} = \alpha\mu(\mathbf{q}) + \phi\kappa(\mathbf{q})\bar{x}$, so that $\bar{x} = \frac{\alpha\mu(\mathbf{q})}{1-\phi\kappa(\mathbf{q})}$. Substituting this expression back into the best response gives the equilibrium effort of a type-ℓ player under the survival profile $\mathbf{q}$:

$$x_\ell(\mathbf{q}) = \alpha q_\ell \big[1 + L(\mathbf{q}) k_\ell\big]. \tag{2}$$

Equation (2) shows how survival and degree affect effort. A higher survival profile $\mathbf{q}$ raises effort through both the private return and expected neighbor effort; a higher own degree k_ℓ raises the return from a given spillover environment. Thus, if $k_r \geq k_s$ and $q_r \geq q_s$, then $x_r(\mathbf{q}) \geq x_s(\mathbf{q})$. The inequality is strict if either $q_r > q_s$, or $q_r = q_s > 0$ and $k_r > k_s$.

The effort ranking determines which classes the planner targets. Taking $\mathbf{x}$ as given, the planner minimizes surviving activity, $\sum_{\ell=1}^{d} m_\ell q_\ell x_\ell$, by allocating enforcement in descending order of effort. Suppose the planner targeted a lower-degree class and left a higher-degree class untargeted. The latter would have a weakly higher survival probability and a strictly greater return from neighbor effort. Thus, by (2), it would exert strictly more effort. So shifting enforcement toward it would lower surviving activity and such an allocation cannot be an equilibrium.

To establish existence and uniqueness, we reduce the equilibrium conditions to a scalar problem. For a conjectured spillover index $L > 0$, let $z_B(L)$ be the cutoff that exhausts the budget, and define $w_B(L) := \frac{z_B(L)}{L}$.

Lemma 2 in Appendix A.2 gives the resulting survival and effort profiles. For a given L, the budget determines a unique $z_B(L)$, and higher values of $1 + Lk_\ell$ imply weakly lower survival. In equilibrium, this survival profile must also generate the conjectured spillover index L. Lemma 2 shows that this consistency condition is strictly monotone, yielding the following result.

Proposition 1 (Equilibrium existence, uniqueness, and cutoff targeting) *Under Assumptions 1 and 2, a type-symmetric pure-strategy equilibrium exists and is unique. In every type-symmetric equilibrium, effort is given by* (2)*, and the social planner's survival profile has a cutoff structure: if $q_s^* < 1$ for some type s, then $q_r^* < 1$ for every higher-degree type $r < s$.*

The targeted classes therefore form an initial block of the degree distribution. Equation (8) also gives $q_\ell^* > 0$ for every ℓ, because $z_B(L^*) > 0$. No class is fully removed. For a targeted class, $q_\ell^* < 1$, and equation (8) gives $x_\ell^* = \alpha z_B(L^*)$. Thus all targeted classes exert the same effort.

Remark 2 (Scope of type-independent neighbor beliefs) Under Assumption 1, all degree classes expect the same effort per neighbor, so a single spillover index summarizes the common spillover environment. If a type-ℓ player instead held type-specific beliefs $\pi_{\cdot|\ell}$, different degree classes would generally face different spillover aggregates. The resulting effort system would be vector-valued, and the scalar characterization in Lemma 2 would require additional restrictions on cross-type mixing. In this sense, Assumption 1 allows us to isolate the roles of the planner's ability to commit to an intervention and of network amplification.

Definition 3 (Interior targeted block) A cutoff survival profile has an interior targeted block $\{1, \dots, \tau\}$ if $0 < q_\ell < 1$ for every $\ell \leq \tau$ and $q_\ell = 1$ for every $\ell > \tau$.

Uniqueness follows from the way the budget and spillover consistency interact. A larger conjectured L raises the return to survival, particularly for high-degree classes. Keeping the budget fixed therefore requires a lower rescaled cutoff $w_B(L)$. At the same time, $\frac{1}{L} + k_\ell$ falls for every class. The consistency map $F_{B,\pi}(L)$ is consequently strictly decreasing in L and cannot admit two distinct equilibrium spillover indices. This argument also supports the comparative statics in Section 5.

For the remainder of the paper, write $\mathbf{q}^{\text{sim}}(B)$ for the unique type-symmetric simultaneous survival profile. Since $B > 0$, at least one class is targeted. Let $T(B) := \{\ell : q_\ell^{\text{sim}}(B) < 1\}$ denote the targeted block and $\tau^{\text{sim}}(B) := \max T(B)$ its cutoff. Proposition 1 gives $T(B) = \{1, \dots, \tau^{\text{sim}}(B)\}$. By Lemma 2(iii), we have $0 < q_\ell^{\text{sim}}(B) < 1$ for every $\ell \in T(B)$, while $q_\ell^{\text{sim}}(B) = 1$ for $\ell \notin T(B)$.

We now ask whether a planner who commits to a targeting policy before players choose effort would select this allocation. The comparison turns on a margin that effort equalization leaves open: intervention changes expected neighbor effort and hence the incentives of players throughout the network.

4 The equalization distortion

4.1 The planner's problem under commitment

Consider a planner who commits to a survival profile before players choose effort. A higher survival probability raises a class's own surviving activity and changes the spillover index $L(\mathbf{q})$, inducing an effort response $\mathbf{x}(\mathbf{q})$ throughout the network.

The simultaneous planner takes effort as given and therefore equalizes effort within the targeted block. The sequential planner also accounts for the response through $L(\mathbf{q})$. Its optimality condition equalizes the full marginal cost of survival across interior classes. To compare the two allocations, we separate the class's own response from the amplification of effort elsewhere in the network.

For a feasible survival profile $\mathbf{q} \in \mathcal{Q}(B)$, equation (2) gives $x_\ell(\mathbf{q}) = \alpha q_\ell[1 + L(\mathbf{q})k_\ell]$. The planner chooses $\mathbf{q}$ anticipating this response. Substituting $\mathbf{x}(\mathbf{q})$ into total surviving activity

yields the sequential objective:

$$W^{\text{seq}}(\mathbf{q}) := \sum_{\ell=1}^{d} m_\ell q_\ell x_\ell(\mathbf{q}) = \alpha \sum_{\ell=1}^{d} m_\ell q_\ell^2 [1 + L(\mathbf{q}) k_\ell]. \tag{3}$$

A sequential optimum solves $\min_{\mathbf{q}\in\mathcal{Q}(B)} W^{\text{seq}}(\mathbf{q})$. Such an optimum exists because $\mathcal{Q}(B)$ is nonempty and compact and W^{seq} is continuous on $\mathcal{Q}(B)$. We do not require uniqueness and write $\mathbf{q}^{\text{seq}}(B)$ for an arbitrary minimizer of W^{seq} over $\mathcal{Q}(B)$.

Define $C(\mathbf{q}) := \sum_{j=1}^{d} m_j k_j q_j^2$ and $\eta(\mathbf{q}) := \frac{\phi C(\mathbf{q})}{1-\phi\kappa(\mathbf{q})}$. The term $\eta(\mathbf{q})$ measures the aggregate amplification generated by the survival profile. The following lemma separates the two components of the sequential marginal cost.

Lemma 1 (Sequential marginal cost of survival) *For every $\mathbf{q} \in \mathcal{Q}(B)$ and every class ℓ,*

$$\frac{1}{m_\ell}\frac{\partial W^{\text{seq}}(\mathbf{q})}{\partial q_\ell} = \underbrace{2x_\ell(\mathbf{q})}_{\textit{direct effect}} + \underbrace{\alpha\eta(\mathbf{q})[1 + L(\mathbf{q})k_\ell]r_\ell}_{\textit{internalized amplification effect}}, \tag{4}$$

where $x_\ell(\mathbf{q}) = \alpha q_\ell[1 + L(\mathbf{q})k_\ell]$ is the effort induced by $\mathbf{q}$, and $r_\ell := \frac{\pi_\ell}{m_\ell}$ is the neighbor-belief weight on class ℓ per unit of population mass.

For class ℓ, the direct component measures the effect of a higher q_ℓ at a fixed spillover index $L(\mathbf{q})$. At a fixed $L(\mathbf{q})$, a higher q_ℓ raises both the effort class ℓ chooses and the share of that effort that survives. By (2), these changes give a marginal cost proportional to class-ℓ effort. The amplification component captures the spillover response. A higher q_ℓ raises $L(\mathbf{q})$, increasing effort in every class with a positive survival probability.

At the simultaneous allocation, effort is equalized across targeted classes. So the direct component in (4) is the same across targeted classes, even though the simultaneous planner takes effort as fixed. Any remaining difference in the sequential marginal return to enforcement within this block comes from amplification. This is the margin on which commitment can improve the allocation.

4.2 The amplification score and suboptimality.

We now evaluate the omitted amplification margin at the simultaneous equilibrium. Let $L^{\text{sim}}(B) := L(\mathbf{q}^{\text{sim}}(B))$ and $\eta^{\text{sim}}(B) := \eta(\mathbf{q}^{\text{sim}}(B))$.

Definition 4 (Amplification score) For each class ℓ, the amplification score at the simultaneous equilibrium is

$$\mathcal{A}_\ell(B) := [1 + L^{\text{sim}}(B)k_\ell]r_\ell.$$

Multiplying $\mathcal{A}_\ell(B)$ by the common factor $\alpha\eta^{\text{sim}}(B)$ gives the amplification component of the sequential marginal cost at the simultaneous allocation. Differences in this score therefore identify differences in the margin that effort equalization leaves unaccounted for.

Proposition 2 (Suboptimality of simultaneous targeting) *Suppose Assumptions 1 and 2 hold. If there exist $a, b \in T(B)$ such that $\mathcal{A}_a(B) > \mathcal{A}_b(B)$, then the simultaneous survival profile $\mathbf{q}^{\text{sim}}(B)$ is not sequentially optimal. In particular, for every sequential optimum $\mathbf{q}^{\text{seq}}(B) \in \arg\min_{\mathbf{q}\in\mathcal{Q}(B)} W^{\text{seq}}(\mathbf{q})$, we have*

$$W^{\text{seq}}(\mathbf{q}^{\text{seq}}(B)) < W^{\text{seq}}\big(\mathbf{q}^{\text{sim}}(B)\big).$$

Proposition 2 compares two classes in the simultaneous targeted block. If their amplification scores differ, shifting a sufficiently small amount of enforcement toward the class with the higher score lowers surviving activity after effort adjusts.[5]

The reason is that, within $T(B)$, the direct components of the sequential marginal cost are equal. Consequently, these components cancel to first order under a budget-neutral reallocation. When amplification scores differ, a marginal increase in enforcement in the class with the higher score reduces activity through the network by more than the same increase in the class with the lower score. Since all classes in $T(B)$ have interior survival probabilities, a sufficiently small budget-neutral shift between them is feasible.

If $\mathcal{A}_a(B) > \mathcal{A}_b(B)$, shifting enforcement from class b toward class a changes the sequential objective at rate $-\alpha\eta^{\text{sim}}(B)\big[\mathcal{A}_a(B) - \mathcal{A}_b(B)\big] < 0$. The negative derivative rules out the simultaneous allocation as a sequential optimum.[6]

4.3 Monotone neighbor-belief weights

We now give a belief restriction under which Proposition 2 ranks the improving reallocations by degree. Recall $r_\ell := \frac{\pi_\ell}{m_\ell}$, the neighbor-belief weight on class ℓ per unit of population mass. We say that neighbor-belief weights are monotone if $\{r_\ell\}_{\ell=1}^d$ is weakly decreasing in ℓ. Since lower indices indicate higher degrees, this means that higher-degree classes receive weakly greater belief weight relative to their population mass.

While this restriction allows for a wider range of beliefs, it holds under exact link-based sampling, which yields

$$\pi_\ell = \frac{k_\ell m_\ell}{\sum_{j=1}^d k_j m_j}, \qquad r_\ell = \frac{\pi_\ell}{m_\ell} = \frac{k_\ell}{\sum_{j=1}^d k_j m_j}.$$

Thus r_ℓ is proportional to k_ℓ, giving higher-degree classes greater neighbor-belief weight per unit mass (Feld, 1991; Newman et al., 2001; Jackson, 2019). Although link-based sampling motivates Assumption 3, we retain the weaker monotonicity restriction. Beliefs also remain independent of a player's own type.

Assumption 3 (Monotone neighbor-belief weights) The sequence $\{r_\ell\}_{\ell=1}^d$ is weakly decreasing in ℓ.

Corollary 1 (Monotone neighbor-belief weights and the equalization distortion) *Suppose Assumptions 1, 2, and 3 hold. If $|T(B)| \geq 2$, then $\mathcal{A}_i(B) > \mathcal{A}_j(B)$ for any $i < j$ in $T(B)$. Consequently, the simultaneous survival profile is not sequentially optimal: $W^{\text{seq}}(\mathbf{q}^{\text{seq}}(B)) < W^{\text{seq}}(\mathbf{q}^{\text{sim}}(B))$.*

The belief restriction makes amplification strictly increase with degree among targeted classes. Whenever the simultaneous policy targets at least two classes, a small shift of enforcement toward a higher-degree class in that block lowers the sequential objective. The two timings thus agree on giving priority to higher-degree classes but differ in the intensity of enforcement within the targeted block.

[5] The same local comparison applies with an increasing convex cost of surviving activity. Across partially targeted classes, the relevant amplification score remains $[1 + L(q)k_\ell]r_\ell$; see Appendix B.3.

[6] The proposition does not identify the sequential optimum or its targeted block. Proposition 4 characterizes the global degree ordering of any sequential optimum.

4.4 When does commitment matter?

Under monotone neighbor-belief weights, commitment strictly reduces total surviving activity whenever the simultaneous equilibrium targets at least two degree classes. We now characterize the budget threshold above which the simultaneous equilibrium targets at least two classes. We then examine how the gain from commitment varies with the budget and the strength of complementarities.

Throughout this subsection, we maintain Assumptions 1, 2, and 3 and assume $d \geq 2$. For $0 < B < n$ and $0 < \phi < 1/\bar{k}$, let $W^{\mathrm{sim}}(B, \phi) := W^{\mathrm{seq}}(\mathbf{q}^{\mathrm{sim}}(B))$ denote total surviving activity at the simultaneous equilibrium, and let $W^{\mathrm{seq}}_{\min}(B, \phi) := \min_{\mathbf{q} \in \mathcal{Q}(B)} W^{\mathrm{seq}}(\mathbf{q})$ denote the minimum of total surviving activity under commitment. The gain from commitment is

$$G(B, \phi) := W^{\mathrm{sim}}(B, \phi) - W^{\mathrm{seq}}_{\min}(B, \phi),$$

which is nonnegative because $\mathbf{q}^{\mathrm{sim}}(B) \in \mathcal{Q}(B)$.

Define the *budget threshold* as

$$B_c(\phi) := \frac{m_1 \phi (k_1 - k_2)}{1 + \phi \left[k_1 - \bar{k} + \pi_1 (k_1 - k_2) \right]}.$$

The threshold satisfies $0 < B_c(\phi) < m_1$, so a budget at or below it is smaller than the mass of class 1.

Proposition 3 (Budget threshold for the gain from commitment) *Suppose Assumptions 1, 2, and 3 hold and $d \geq 2$. The simultaneous equilibrium targets exactly one class if and only if $B \leq B_c(\phi)$. In this case, the sequential optimum is unique and coincides with the simultaneous survival profile:*

$$\mathbf{q}^{\mathrm{seq}}(B) = \mathbf{q}^{\mathrm{sim}}(B) = \left(1 - \frac{B}{m_1}, 1, \ldots, 1 \right).$$

Consequently, $G(B, \phi) = 0$ if $B \leq B_c(\phi)$, and $G(B, \phi) > 0$ if $B > B_c(\phi)$.

At or below $B_c(\phi)$, both planners allocate their entire budget to class 1. Above the threshold, the simultaneous planner targets at least two classes and equalizes effort across them. The sequential planner also anticipates how enforcement changes network spillovers. Because this amplification effect is larger for higher-degree targeted classes, a sufficiently small budget-neutral shift of enforcement from a lower-degree targeted class to a higher-degree one reduces total surviving activity after effort adjusts. The gain from commitment is thus strictly positive.

Corollary 2 (The gain from commitment and the enforcement budget) *Suppose Assumptions 1, 2, and 3 hold and $d \geq 2$. Then $\lim_{B \to n^-} G(B, \phi) = 0$. Moreover, there exists $B^* \in (B_c(\phi), n)$ such that*

$$G(B^*, \phi) = \max_{0 < B < n} G(B, \phi) > 0.$$

Hence $G(B, \phi)$ is nonmonotone in B.

For budgets at or below $B_c(\phi)$, both planners target only class 1 and choose the same policy, so the gain from commitment is zero. As $B \to n^-$, total surviving activity converges to zero

under both policies, so the gain also converges to zero. The gain from commitment is therefore largest at an intermediate budget.

Furthermore, the threshold increases with respect to the strength of complementarities, since

$$B_c'(\phi) = \frac{m_1(k_1 - k_2)}{\left\{1 + \phi\left[k_1 - \bar{k} + \pi_1(k_1 - k_2)\right]\right\}^2} > 0.$$

Stronger complementarities thus expand the range of budgets for which the two policies coincide. As $\phi \to (1/\bar{k})^-$, the threshold increases to

$$\lim_{\phi \to (1/\bar{k})^-} B_c(\phi) = \frac{m_1(k_1 - k_2)}{k_1 + \pi_1(k_1 - k_2)}.$$

If the budget is below this limit, then $B \leq B_c(\phi)$ for all ϕ sufficiently close to $1/\bar{k}$, and the gain from commitment is zero for these values of ϕ.

Corollary 3 (The gain from commitment and complementarities) *Suppose Assumptions 1 and 3 hold, $d \geq 2$, and the budget satisfies*

$$0 < B < \frac{m_1(k_1 - k_2)}{k_1 + \pi_1(k_1 - k_2)}.$$

Define

$$\phi_c := \frac{B}{m_1(k_1 - k_2) - B\left[k_1 - \bar{k} + \pi_1(k_1 - k_2)\right]}.$$

Then $0 < \phi_c < 1/\bar{k}$, $G(B, \phi) > 0$ for $0 < \phi < \phi_c$, and $G(B, \phi) = 0$ for $\phi_c \leq \phi < 1/\bar{k}$. Moreover, $\lim_{\phi \to 0^+} G(B, \phi) = 0$, and there exists $\phi^* \in (0, \phi_c)$ such that

$$G(B, \phi^*) = \max_{0 < \phi < 1/\bar{k}} G(B, \phi) > 0.$$

Hence $G(B, \phi)$ is nonmonotone in ϕ.

As $\phi \to 0^+$, network complementarities vanish, so the gain from anticipating the network response to enforcement converges to zero. As ϕ increases, effort becomes more sensitive to degree, and the simultaneous planner concentrates enforcement on weakly fewer classes. For $\phi \geq \phi_c$, it allocates its entire budget to class 1, and its policy coincides with the sequential optimum. Hence, stronger complementarities need not raise the gain from commitment: for a budget satisfying the condition in Corollary 3, they eventually reduce the gain to zero.

4.5 Sequential targeting and key-player removal

The local comparison in Proposition 2 establishes suboptimality without solving the sequential problem. We now characterize the structure of any global sequential optimum. Under monotone neighbor-belief weights, the sequential planner also gives priority to higher-degree classes and equalizes the full marginal cost of survival among partially targeted classes.

Proposition 4 (Degree ordering of sequential targeting) *Suppose Assumptions 1, 2, and 3 hold. Let* $\mathbf{q}^{\text{seq}}(B)$ *be any sequential optimum. Then*

$$q_1^{\text{seq}}(B) \leq q_2^{\text{seq}}(B) \leq \cdots \leq q_d^{\text{seq}}(B).$$

Thus higher-degree classes face weakly higher removal probabilities. Moreover, there exist cutoffs $0 \leq h^{\text{seq}} \leq \tau^{\text{seq}} \leq d$ *such that* $q_\ell^{\text{seq}}(B) = 0$ *for* $\ell \leq h^{\text{seq}}$, $0 < q_\ell^{\text{seq}}(B) < 1$ *for* $h^{\text{seq}} < \ell \leq \tau^{\text{seq}}$, *and* $q_\ell^{\text{seq}}(B) = 1$ *for* $\ell > \tau^{\text{seq}}$. *Within any nonempty partially targeted block, survival probabilities are strictly increasing as degree falls.*

Proposition 4 separates the sequential allocation into three possible regions under Assumption 3. The highest-degree classes may be fully removed, $q_\ell^{\text{seq}} = 0$; an intermediate block may be partially targeted, $0 < q_\ell^{\text{seq}} < 1$; and the lowest-degree classes may be left untargeted, $q_\ell^{\text{seq}} = 1$. Within the partially targeted block, survival increases as degree falls.

Why can commitment policy make full removal optimal? In the simultaneous game, a class with $q_\ell = 0$ exerts $x_\ell = 0$. Taking effort as fixed, the planner can then lower activity by transferring enforcement to a class with positive effort. The sequential planner anticipates that restoring survival also restores the removed class's incentive to invest. Even at $q_\ell = 0$, Lemma 1 gives a positive marginal cost of survival: $\alpha\eta^{\text{seq}}[1 + L^{\text{seq}}k_\ell]r_\ell > 0$. Full removal can therefore satisfy the sequential optimality condition when this marginal cost is weakly above the shadow value of the budget. Under Assumption 3, the amplification component is largest for high-degree classes, so any fully removed classes must lie at the top of the degree distribution.

This gives a class-level comparison with key-player removal. A policy that fully removes the highest-degree classes first is consistent with the degree ordering characterized here, but Proposition 4 does not generally establish full removal of every targeted class. Rather, the planner compares the full marginal costs of survival. As enforcement increases on a class, its own effort falls, and an interior survival probability may satisfy the marginal condition before the class is fully removed. The planner then also targets other classes. The belief ordering in Assumption 3 keeps this allocation ordered by degree, while allowing partial targeting (Ballester et al., 2006; Galeotti et al., 2020; Li et al., 2022).

The following example illustrates how much total surviving activity differs when the social planner can commit to a targeting policy.

4.6 An example

Example 1 (The size of the equalization distortion) *Normalize* $\alpha = 1$, *consider four degree classes with* $(k_1, k_2, k_3, k_4) = (10, 6, 4, 3)$ *and masses* $(m_1, m_2, m_3, m_4) = (50, 80, 120, 150)$. *Suppose beliefs about future neighbors are link-based:* $\pi_\ell = \frac{k_\ell m_\ell}{\sum_{j=1}^4 k_j m_j}$. *Rounding to two decimal places gives* $\boldsymbol{\pi} = (0.26, 0.25, 0.25, 0.24)$. *The planner's budget is* $B = 60$, *covering 15 percent of population mass. We vary* ϕ *below the stability bound* $\phi\bar{k} < 1$, *where the belief-weighted mean degree is* $\bar{k} = 5.84$ *after rounding. Link-based neighbor beliefs satisfy Assumption 3 because* $\frac{\pi_\ell}{m_\ell} = \frac{k_\ell}{\sum_{j=1}^4 k_j m_j}$.

Corollary 1 implies a strict gain from commitment in this example whenever the simultaneous policy targets more than one class. Table 1 reports the simultaneous equilibrium and the sequential optimum.

The simultaneous policy equalizes effort within the targeted block. At $\phi = 0.15$, *for instance, the block is* $\{1, 2, 3\}$, *with* $\mathbf{q}^{\text{sim}} = (0.50, 0.71, 0.90, 1.00)$ *and* $x_1^{\text{sim}} = x_2^{\text{sim}} = x_3^{\text{sim}} = 1.94$. *At this*

allocation, the amplification components $\eta^{\text{sim}}\mathcal{A}_\ell$ *are* $(9.18, 3.87, 2.04, 1.32)$*. The highest-degree class therefore has a much larger amplification component despite exerting the same effort as the other targeted classes. The sequential planner accounts for this difference, generating the gain reported in Table 1.*

Table 1: Gains from commitment

ϕ	$\phi\bar{k}$	T^{sim}	$\mathbf{q}^{\text{sim}}$	$\mathbf{q}^{\text{seq}}$	W^{sim}	$W^{\text{seq}}_{\min}$	Reduction in surviving activity (%)
0.05	0.29	$\{1,2,3,4\}$	$(0.69, 0.80, 0.87, 0.91)$	$(0.57, 0.77, 0.89, 0.95)$	358.94	357.26	0.47
0.08	0.47	$\{1,2,3,4\}$	$(0.62, 0.77, 0.88, 0.95)$	$(0.41, 0.73, 0.93, 1.00)$	416.06	408.85	1.73
0.10	0.58	$\{1,2,3,4\}$	$(0.58, 0.75, 0.88, 0.97)$	$(0.31, 0.72, 0.98, 1.00)$	463.87	449.29	3.14
0.12	0.70	$\{1,2,3,4\}$	$(0.54, 0.73, 0.88, 0.99)$	$(0.21, 0.74, 1.00, 1.00)$	522.72	495.72	5.17
0.14	0.82	$\{1,2,3\}$	$(0.51, 0.72, 0.89, 1.00)$	$(0.13, 0.80, 1.00, 1.00)$	599.67	550.84	8.14
0.15	0.88	$\{1,2,3\}$	$(0.50, 0.71, 0.90, 1.00)$	$(0.08, 0.83, 1.00, 1.00)$	648.71	582.12	10.26
0.16	0.93	$\{1,2,3\}$	$(0.49, 0.71, 0.91, 1.00)$	$(0.02, 0.86, 1.00, 1.00)$	706.69	616.04	12.83

Notes: The table holds fixed the degree distribution, class masses, link-based neighbor beliefs, and budget. Here $W^{\text{sim}} := W^{\text{seq}}(\mathbf{q}^{\text{sim}})$, and $W^{\text{seq}}_{\min} := \min_{\mathbf{q}\in\mathcal{Q}(B)} W^{\text{seq}}(\mathbf{q})$. The final column reports $\frac{W^{\text{sim}} - W^{\text{seq}}_{\min}}{W^{\text{sim}}} \times 100$. Appendix B.2 verifies the reported sequential profiles and values.

4.7 Alternative specification: Surviving-neighbor spillovers

In the baseline model, enforcement changes expected neighbor effort through players' investment decisions. We now allow enforcement also to remove the neighbor activity that generates complementarities. The same comparison between effort equalization and amplification applies, but the condition for a strict improvement changes.

Consider an alternative specification in which enforcement also enters the spillover aggregate. Conditional on their degree types and the survival profile $\mathbf{q}$, a type-ℓ player's activity and a type-j neighbor's activity both survive with probability $q_\ell q_j$. A type-ℓ player's expected payoff is

$$U_\ell = \alpha q_\ell x_\ell - \frac{1}{2}x_\ell^2 + \phi k_\ell q_\ell x_\ell \bar{y}, \qquad \bar{y} := \sum_{j=1}^{d} \pi_j q_j x_j.$$

Here $\bar{y}$ is expected surviving-neighbor effort. Compared with the baseline in (1), the alternative payoff specification allows enforcement to affect incentives through two channels. It lowers a player's own return to effort, and it directly reduces the neighbor activity that enters the complementarity term.

Lemma 3 in Appendix B.1 gives the induced effort response:

$$x_\ell(\mathbf{q}) = \alpha q_\ell[1 + \widetilde{L}(\mathbf{q})k_\ell], \qquad \widetilde{L}(\mathbf{q}) := \frac{\phi \sum_{j=1}^{d} \pi_j q_j^2}{1 - \phi \sum_{j=1}^{d} \pi_j k_j q_j^2}.$$

The expression has the same form as (2), but the index now depends on q_j^2 rather than q_j. Neighbor j's contribution to expected surviving-neighbor effort depends on both her survival probability and the effort she chooses given that probability.

The social planner's problem remains unchanged. Given $\mathbf{x}$, it chooses $\mathbf{q} \in \mathcal{Q}(B)$ to minimize $\sum_{\ell=1}^{d} m_\ell q_\ell x_\ell$. All partially targeted classes must then exert the same effort in a simultaneous equilibrium. Proposition 7 establishes the same cutoff targeting result and rules out full removal of any type in equilibrium. Moreover, Proposition 8 shows uniqueness of type-symmetric equilibrium under Assumptions 1, 2, and 3.

The sequential marginal condition, however, changes under this specification. Let $\widetilde{W}$ denote the total surviving activity. Lemma 4 in Appendix B.1 gives, for every $\mathbf{q} \in \mathcal{Q}(B)$,

$$\frac{1}{m_\ell}\frac{\partial \widetilde{W}(\mathbf{q})}{\partial q_\ell} = \underbrace{2x_\ell(\mathbf{q})}_{\text{direct effect}} + \underbrace{2\widetilde{\eta}(\mathbf{q})x_\ell(\mathbf{q})r_\ell}_{\text{internalized amplification effect}},$$

where $r_\ell := \frac{\pi_\ell}{m_\ell}$, $\widetilde{\eta}(\mathbf{q}) := \frac{\phi \widetilde{C}(\mathbf{q})}{1-\phi\widetilde{\kappa}(\mathbf{q})}$, $\widetilde{C}(\mathbf{q}) := \sum_{j=1}^{d} m_j k_j q_j^2$, and $\widetilde{\kappa}(\mathbf{q}) := \sum_{j=1}^{d} \pi_j k_j q_j^2$.

The direct component for class ℓ is the same as in the baseline model. By comparison with Lemma 1, the amplification component is now proportional to $x_\ell(\mathbf{q})r_\ell$, rather than $[1+L(\mathbf{q})k_\ell]r_\ell$. Thus, among classes that exert the same effort, any difference in amplification comes from differences in relative neighbor-belief weights.

Proposition 5 (Suboptimality of simultaneous targeting under surviving-neighbor spillovers) *Let $(\mathbf{q}^s, \mathbf{x}^s)$ be any type-symmetric simultaneous equilibrium under the surviving-neighbor specification, and let $T^s := \{\ell : q_\ell^s < 1\}$ be its targeted block. If there exist $i, j \in T^s$ such that $r_i > r_j$, then $\mathbf{q}^s$ is not sequentially optimal under this specification. Hence, for any sequential optimum $\widetilde{\mathbf{q}}^{\text{seq}}(B)$,*

$$\widetilde{W}\big(\widetilde{\mathbf{q}}^{\text{seq}}(B)\big) < \widetilde{W}(\mathbf{q}^s).$$

This stronger condition has a simple explanation. In the baseline model, the amplification score is $\mathcal{A}_\ell(B) = [1+L^{\text{sim}}(B)k_\ell]r_\ell$, so degree directly affects the omitted margin. Here, however, the amplification component is proportional to $x_\ell(\mathbf{q})r_\ell$. Simultaneous targeting equalizes x_ℓ, so that relative neighbor-belief weights are the source of any within-block difference. If $r_i > r_j$ for two targeted classes, a sufficiently small budget-neutral shift of enforcement from class j to class i lowers the sequential objective. Under link-based neighbor beliefs, $r_\ell = \frac{k_\ell}{\sum_{j=1}^{d} k_j m_j}$. Thus $r_i > r_j$ whenever $k_i > k_j$. Under these beliefs, targeting at least two classes is again sufficient for a strict sequential improvement, i.e., positive gain from commitment.

Furthermore, for this alternative specification, under population-weighted neighbor beliefs, i.e., $\pi_\ell = \frac{m_\ell}{n}$, Appendix B.1 shows that the simultaneous and sequential policies coincide for every $B \in (0, n)$ (Proposition 9). This is because under population-weighted neighbor beliefs, each class receives the same relative weight in expected surviving-neighbor effort as in the planner's objective. Thus, at any given survival profile $\mathbf{q}$, the sequential marginal cost of survival per player in class ℓ equals that class's effort $x_\ell(\mathbf{q})$ multiplied by a positive factor that is common to all classes. Accounting for the network response therefore preserves the relative returns to enforcement across classes, so the planners choose the same allocation with and without commitment. Hence, strategic complementarities alone need not generate a gain from commitment.

5 Comparative statics analysis

The degree-based local improvement requires at least two classes in the simultaneous targeted block. We now study how this block changes with the enforcement budget, the strength of

complementarities, and neighbor beliefs.

Proposition 6 (Comparative statics of simultaneous targeting) *Suppose Assumptions 1 and 2 hold. Let $L^*(B,\phi,\pi)$, $x_\ell^*(B,\phi,\pi)$, and $\tau^*(B,\phi,\pi)$ denote the simultaneous-equilibrium spillover index, type-ℓ effort, and targeting cutoff.*

(i) If $B' > B$, holding (ϕ,π) fixed, then $L^*(B',\phi,\pi) < L^*(B,\phi,\pi)$, $x_\ell^*(B',\phi,\pi) < x_\ell^*(B,\phi,\pi)$ for every ℓ, and $\tau^*(B',\phi,\pi) \geq \tau^*(B,\phi,\pi)$.

(ii) If $\phi'' > \phi'$, holding (B,π) fixed, and both values satisfy Assumption 2, then $L^*(B,\phi'',\pi) > L^*(B,\phi',\pi)$, $x_\ell^*(B,\phi'',\pi) > x_\ell^*(B,\phi',\pi)$ for every ℓ, and $\tau^*(B,\phi'',\pi) \leq \tau^*(B,\phi',\pi)$.

(iii) Fix (B,ϕ). Let π and π' both satisfy Assumption 1, and suppose Assumption 2 holds under both belief distributions. If the distribution of neighbor degree under π' first-order stochastically dominates that under π, so that $\sum_{\ell=1}^{s}\pi'_\ell \geq \sum_{\ell=1}^{s}\pi_\ell$ for every $s = 1,\ldots,d-1$, then $L^*(B,\phi,\pi') \geq L^*(B,\phi,\pi)$, $x_\ell^*(B,\phi,\pi') \geq x_\ell^*(B,\phi,\pi)$ for every ℓ, and $\tau^*(B,\phi,\pi') \leq \tau^*(B,\phi,\pi)$.

A larger budget B reduces equilibrium effort in every class and lowers the spillover index, while weakly expanding targeting toward lower-degree classes. Under monotone neighbor-belief weights, this expansion can make the local distortion relevant: once a second class enters the targeted block, unequal amplification permits a sequential improvement even though effort is equalized.

Stronger complementarities ϕ raise spillovers and effort while weakly reducing the number of targeted classes. A shift of neighbor beliefs π toward higher-degree types weakly raises spillovers and effort and weakly contracts the targeted block. These comparative statics describe the set of classes over which the local comparison operates.

6 Conclusion

This paper studies how commitment to an enforcement policy changes targeting in a network game. A planner moving simultaneously with players takes effort as given and equalizes it across the targeted classes. A planner who commits before effort is chosen also accounts for network amplification. Under monotone neighbor-belief weights, this amplification is stronger for higher-degree classes, so the simultaneous allocation can be improved whenever it targets more than one class. Commitment preserves the degree ordering of targeting and can change enforcement intensity and can make full removal optimal in the baseline specification.

Under the same belief restriction, our baseline model characterizes an exact budget threshold. At or below this threshold, both simultaneous and sequential planners devote their entire budget to the highest-degree class and choose the same policy, while above it, commitment strictly reduces total surviving activity relative to the simultaneous policy. Near complete enforcement, network spillovers vanish so that the gain from commitment converges to zero. As a result, commitment is most valuable at intermediate budgets relative to the size of the criminal population. Also, for a sufficiently small fixed budget, stronger complementarities can eliminate the gain by making the simultaneous planner devote its entire budget to the highest-degree class, so the two policies coincide.

Our analysis holds the degree classes and enforcement technology fixed. One extension would allow the planner to use partially informative signals within a class. Another extension

would allow players' degree types to respond to anticipated enforcement implemented by the social planner. Both of these extensions would help relate the targeting comparison to settings such as platform content moderation, where observable account features provide information about connectedness and can also respond to policy.

A further extension would allow for partial commitment. For example, a planner might be able to commit with some probability or consider the option to revise the enforcement after receiving a signal. Such a model would clarify how the value of commitment depends on the information available when policy is revised and the cost of restricting that revision.

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Appendix

A Proofs of the results in the main text

A.1 Preliminary results for Section 3

Reduced-form effort. Fix a feasible survival profile $\mathbf{q}$. Given expected neighbor effort $\bar{x}$, a type-ℓ player solves

$$\max_{x_\ell \geq 0} \left\{ \alpha q_\ell x_\ell - \frac{1}{2} x_\ell^2 + \phi k_\ell q_\ell x_\ell \bar{x} \right\}. \tag{5}$$

The objective in (5) is strictly concave in x_ℓ. Its unique maximizer is therefore $x_\ell = q_\ell(\alpha + \phi k_\ell \bar{x})$. Under type symmetry, $\bar{x} = \sum_{\ell=1}^{d} \pi_\ell x_\ell$, so $\bar{x} = \alpha\mu(\mathbf{q}) + \phi\kappa(\mathbf{q})\bar{x}$. By Assumption 2, $1 - \phi\kappa(\mathbf{q}) > 0$. Hence $\bar{x} = \frac{\alpha\mu(\mathbf{q})}{1-\phi\kappa(\mathbf{q})}$. Substituting this value of $\bar{x}$ into $x_\ell = q_\ell(\alpha + \phi k_\ell \bar{x})$ gives

$$x_\ell(\mathbf{q}) = \alpha q_\ell \big[1 + L(\mathbf{q}) k_\ell\big]. \tag{6}$$

This expression is well defined. Since $q_\ell \in [0,1]$ for every ℓ, $\kappa(\mathbf{q}) \leq \sum_{\ell=1}^{d} \pi_\ell k_\ell = \bar{k}$. By Assumption 2, $1 - \phi\kappa(\mathbf{q}) \geq 1 - \phi\bar{k} > 0$. Moreover, since $B < n$, some type survives with positive probability. Since $\pi_\ell > 0$ for every ℓ, this implies $\mu(\mathbf{q}) > 0$, and hence $L(\mathbf{q}) > 0$.

Effort ordering. Suppose $k_r \geq k_s$ and $q_r \geq q_s$. By (6),

$$x_r(\mathbf{q}) - x_s(\mathbf{q}) = \alpha(q_r - q_s) + \alpha L(\mathbf{q})(k_r q_r - k_s q_s).$$

The first term is nonnegative because $q_r \geq q_s$. The second term is nonnegative because $k_r q_r \geq k_s q_s$ and $L(\mathbf{q}) > 0$, where the latter follows from Definition 2, Assumption 2, and $B < n$. Hence $x_r(\mathbf{q}) \geq x_s(\mathbf{q})$. The inequality is strict if $q_r > q_s$, or if $q_r = q_s > 0$ and $k_r > k_s$.

Social planner best response. Fix a type-symmetric effort profile $\mathbf{x}$. The social planner solves

$$\min_{\mathbf{q} \in [0,1]^d} \sum_{\ell=1}^{d} m_\ell q_\ell x_\ell \quad \text{subject to} \quad \sum_{\ell=1}^{d} m_\ell q_\ell = n - B.$$

Since $B \in (0, n)$, the constraint set is nonempty and compact. Since the objective and constraints here are linear with respect to $\mathbf{q}$, the first-order conditions would be necessary and sufficient. Hence there exists a multiplier λ such that, for each ℓ,

$$x_\ell > \lambda \Rightarrow q_\ell = 0, \qquad x_\ell < \lambda \Rightarrow q_\ell = 1, \qquad 0 < q_\ell < 1 \Rightarrow x_\ell = \lambda.$$

Thus the planner assigns enforcement first to classes with higher effort. If $x_1 \geq \cdots \geq x_d$, there is an optimal allocation in which $q_\ell < 1$ exactly when $\ell \leq \tau$, for some $\tau \in \{1, \ldots, d\}$.

A.2 Lemma 2

Lemma 2 (Scalar characterization of simultaneous equilibrium) *Fix $B \in (0, n)$. For every $L > 0$, let $z_B(L)$ be the unique solution to $\sum_{\ell=1}^{d} m_\ell \min\left\{1, \frac{z}{1+Lk_\ell}\right\} = n - B$, and define*

$$w_B(L) := \frac{z_B(L)}{L}, \qquad c_B(L) := \frac{z_B(L) - 1}{L}, \qquad F_{B,\pi}(L) := \sum_{\ell=1}^{d} \pi_\ell \min\left\{w_B(L), \frac{1}{L} + k_\ell\right\}.$$

Under Assumptions 1 and 2:

(i) The cutoff $z_B(L)$, and hence $w_B(L)$, is continuous in L. For fixed B, $w_B(L)$ is strictly decreasing in L, $z_B(L)$ is strictly increasing in L, and $c_B(L)$ is weakly increasing in L. For fixed L, $z_B(L)$, $w_B(L)$, and $c_B(L)$ are strictly decreasing in B.

(ii) The scalar map $F_{B,\pi}$ is continuous and strictly decreasing on $(0, \infty)$, with

$$\lim_{L \downarrow 0} F_{B,\pi}(L) = +\infty, \qquad \limsup_{L \to \infty} F_{B,\pi}(L) \leq \bar{k} < \frac{1}{\phi}.$$

(iii) A pair $(\mathbf{q}, \mathbf{x})$ is a type-symmetric simultaneous equilibrium if and only if its spillover index $L > 0$ solves

$$F_{B,\pi}(L) = \frac{1}{\phi}, \tag{7}$$

and

$$q_\ell = \min\left\{1, \frac{z_B(L)}{1 + Lk_\ell}\right\}, \qquad x_\ell = \alpha \min\{z_B(L), 1 + Lk_\ell\}. \tag{8}$$

Consequently, equation (7) has exactly one solution L^*, and it uniquely determines the simultaneous equilibrium.

(iv) If $B' > B$, then $F_{B',\pi}(L) < F_{B,\pi}(L)$ for every fixed $L > 0$.

(v) If the distribution of neighbor degree under π' first-order stochastically dominates that under π, so that $\sum_{\ell=1}^{s} \pi'_\ell \geq \sum_{\ell=1}^{s} \pi_\ell$ for every $s = 1, \ldots, d-1$, then $F_{B,\pi'}(L) \geq F_{B,\pi}(L)$ for every fixed $L > 0$.

Proof of Lemma 2: For $L > 0$, define $H_L(z) := \sum_{\ell=1}^{d} m_\ell \min\left\{1, \frac{z}{1+Lk_\ell}\right\}$. The function H_L is continuous, $H_L(0) = 0$, and $H_L(z) = n$ for all sufficiently large z. It is strictly increasing whenever its value is below n. Since $n - B \in (0, n)$, there is a unique $z_B(L) > 0$ such that $H_L(z_B(L)) = n - B$.

Step 1. Continuity and cutoff monotonicity. We now establish that the map $z_B(L)$ is continuous. To see this, let us take $L_s \to L > 0$ and denote $z_s := z_B(L_s)$. For all sufficiently large s, L_s remain in a compact subset of $(0, \infty)$. Since $n - B = H_{L_s}(z_s) \leq nz_s$, we have $z_s \geq \frac{n-B}{n}$. Since $H_{L_s}(z) = n$ for all $z \geq \max_\ell\{1 + L_s k_\ell\}$, $H_{L_s}(z_s) = n - B < n$ implies $z_s < \max_\ell\{1 + L_s k_\ell\}$. Thus $c \leq z_s \leq C$ for all s, for some constants $0 < c \leq C < \infty$. Any convergent subsequence $z_{s_j} \to \bar{z}$ satisfies, by continuity of $(L, z) \mapsto H_L(z)$, $H_L(\bar{z}) = n - B$. Uniqueness of the solution implies $\bar{z} = z_B(L)$. Hence every convergent subsequence has the same limit, so $z_s \to z_B(L)$. Therefore z_B, and thus $w_B(L) = \frac{z_B(L)}{L}$, is continuous.

For fixed L, increasing B lowers the required aggregate survival $n - B$. Since H_L is strictly increasing below n, $z_B(L)$ is strictly decreasing in B. With L fixed, the same is true of $w_B(L) = \frac{z_B(L)}{L}$ and $c_B(L) = \frac{z_B(L)-1}{L}$.

Now fix B and take $L' > L$. Define $h_\ell(L) := \frac{1}{L} + k_\ell$. The budget equation can be written as $\sum_{\ell=1}^d m_\ell \min\left\{1, \frac{w_B(L)}{h_\ell(L)}\right\} = n - B$. Since $h_\ell(L') < h_\ell(L)$ for every ℓ, holding $w = w_B(L)$ fixed weakly raises every survival term and strictly raises at least one, because aggregate survival is strictly below n. Hence the budget at L' requires $w_B(L') < w_B(L)$. Holding instead $z = z_B(L)$ fixed, every term $\min\{1, \frac{z}{1+Lk_\ell}\}$ weakly falls when L rises to L', and at least one falls strictly. Thus $z_B(L') > z_B(L)$. Finally write $z = 1 + Lc$ and define $S(L, c) := \sum_{\ell=1}^d m_\ell \min\left\{1, \frac{1+Lc}{1+Lk_\ell}\right\}$. For fixed c, if $k_\ell \le c$ the class-ℓ term equals one, whereas if $k_\ell > c$ it is strictly decreasing in L. Thus $S(L, c)$ is weakly decreasing in L. Since $S(L, c_B(L)) = n - B$, it follows that $c_B(L') \ge c_B(L)$. This proves part (i).

Step 2. The scalar map. Because w_B is continuous, $F_{B,\pi}$ is continuous. If $L' > L$, then both $w_B(L') < w_B(L)$ and $h_\ell(L') < h_\ell(L)$ for every ℓ. Therefore $\min\{w_B(L'), h_\ell(L')\} < \min\{w_B(L), h_\ell(L)\}$ for every ℓ. Since $\pi_\ell > 0$, $F_{B,\pi}(L') < F_{B,\pi}(L)$.

We next establish the boundary behavior. As $L \downarrow 0$, $H_L(1) = \sum_{\ell=1}^d \frac{m_\ell}{1+Lk_\ell} \longrightarrow n$. Since $n - B < n$, for all sufficiently small L we have $H_L(1) > n - B$, and therefore $z_B(L) < 1$. All classes are then interior in the budget equation, so $z_B(L) \sum_{\ell=1}^d \frac{m_\ell}{1+Lk_\ell} = n - B$. Hence $z_B(L) \to \frac{n-B}{n} > 0$. Moreover $z_B(L) < 1 < 1 + Lk_\ell$, so $w_B(L) < h_\ell(L)$ for every class. Thus $F_{B,\pi}(L) = w_B(L) = \frac{z_B(L)}{L} \longrightarrow +\infty$. At the other boundary, $F_{B,\pi}(L) \le \sum_{\ell=1}^d \pi_\ell h_\ell(L) = \frac{1}{L} + \bar{k}$, so $\limsup_{L\to\infty} F_{B,\pi}(L) \le \bar{k} < \frac{1}{\phi}$ by Assumption 2. This proves part (ii).

Step 3. Equilibrium equivalence. First take any type-symmetric simultaneous equilibrium $(\mathbf{q}, \mathbf{x})$, and let $L := L(\mathbf{q}) > 0$. We first rule out complete removal. Suppose $q_r = 0$ for some class r. Since $\sum_\ell m_\ell q_\ell = n - B > 0$, there exists s with $q_s > 0$. By the reduced-form effort equation, $x_r = 0 < x_s$. For sufficiently small $\varepsilon > 0$, raising q_r by $\frac{\varepsilon}{m_r}$ and lowering q_s by $\frac{\varepsilon}{m_s}$ preserves the budget and changes the planner's fixed-effort objective by $\varepsilon(x_r - x_s) < 0$, a contradiction. Hence $q_\ell > 0$ for every ℓ.

Because $B > 0$, at least one class has $q_\ell < 1$. The planner's linear program therefore has a multiplier $\lambda > 0$ such that

$$x_\ell = \lambda \quad \text{if } 0 < q_\ell < 1, \qquad x_\ell \le \lambda \quad \text{if } q_\ell = 1.$$

Using $x_\ell = \alpha q_\ell(1 + Lk_\ell)$ and setting $z := \frac{\lambda}{\alpha}$, we obtain $q_\ell = \min\left\{1, \frac{z}{1+Lk_\ell}\right\}$. The budget constraint therefore implies $z = z_B(L)$. Consequently $x_\ell = \alpha \min\{z_B(L), 1 + Lk_\ell\}$. Spillover consistency requires $L = \frac{\phi\mu(\mathbf{q})}{1-\phi\kappa(\mathbf{q})}$, or equivalently $L = \phi \sum_{\ell=1}^d \pi_\ell q_\ell(1 + Lk_\ell)$. Since $1 + Lk_\ell = Lh_\ell(L)$ and $q_\ell = \min\left\{1, \frac{w_B(L)}{h_\ell(L)}\right\}$, we have $q_\ell(1 + Lk_\ell) = L \min\{w_B(L), h_\ell(L)\}$. Dividing the consistency condition by $L > 0$ gives $F_{B,\pi}(L) = \frac{1}{\phi}$. Thus every equilibrium satisfies the scalar condition and the profile formula in part (iii).

Conversely, suppose $L > 0$ satisfies $F_{B,\pi}(L) = \frac{1}{\phi}$, and define $\mathbf{q}$ and $\mathbf{x}$ by (8). The definition of $z_B(L)$ makes $\mathbf{q}$ feasible. Multiplying the scalar condition by L and reversing the preceding algebra gives $L(1 - \phi\kappa(\mathbf{q})) = \phi\mu(\mathbf{q})$. Assumption 2 gives $1 - \phi\kappa(\mathbf{q}) > 0$, so $L = L(\mathbf{q})$. Hence $\mathbf{x}$ is exactly the players' type-symmetric best-response profile under $\mathbf{q}$.

It remains to verify planner optimality. Let $\lambda := \alpha z_B(L)$. If $q_\ell < 1$, then $q_\ell = \frac{z_B(L)}{1+Lk_\ell}$, so $x_\ell = \lambda$. If $q_\ell = 1$, then $z_B(L) \ge 1 + Lk_\ell$, so $x_\ell = \alpha(1 + Lk_\ell) \le \lambda$. Since $z_B(L) > 0$, no lower

bound binds. These are the necessary and sufficient optimality conditions for the planner's linear program. Therefore $(\mathbf{q}, \mathbf{x})$ is a simultaneous equilibrium.

By part (ii), $F_{B,\pi}$ is continuous and strictly decreasing, starts above $\frac{1}{\phi}$, and eventually lies below $\frac{1}{\phi}$. Hence there exists exactly one $L^* > 0$ satisfying (7). The formulas in (8) then uniquely determine $(\mathbf{q}^*, \mathbf{x}^*)$. This proves part (iii).

Step 4. Parameter shifts of the scalar map. If $B' > B$, part (i) gives $w_{B'}(L) < w_B(L)$. Hence every term in $F_{B',\pi}(L)$ is weakly smaller than the corresponding term in $F_{B,\pi}(L)$. Because aggregate survival is below n, at least one class satisfies $w_B(L) < h_\ell(L)$, and lowering $w_B(L)$ strictly lowers that term. Since $\pi_\ell > 0$, $F_{B',\pi}(L) < F_{B,\pi}(L)$, which proves part (iv).

Finally, for fixed (B, L), the sequence $\min\left\{w_B(L), \frac{1}{L} + k_\ell\right\}$ is weakly decreasing in ℓ, because $k_1 > \cdots > k_d$. A first-order stochastic shift of π toward lower indices, equivalently toward higher-degree classes, therefore weakly raises its expectation. Thus $F_{B,\pi'}(L) \geq F_{B,\pi}(L)$, proving part (v). □

A.3 Proof of Proposition 1

Existence follows directly from Lemma 2(ii)–(iii): the continuous, strictly decreasing scalar map crosses $\frac{1}{\phi}$ exactly once, and the resulting profile in (8) is a simultaneous equilibrium.

It remains only to record the cutoff implication. If $q_s^* < 1$, then $z_B(L^*) < 1 + L^* k_s$. For any higher-degree class $r < s$, $k_r > k_s$, so $z_B(L^*) < 1 + L^* k_r$, and therefore $q_r^* < 1$ by (8). Thus targeted classes form an initial segment of the degree distribution. The effort formula (2) holds by construction.

Uniqueness: By Lemma 2(iii), every type-symmetric simultaneous equilibrium corresponds to a solution of $F_{B,\pi}(L) = \frac{1}{\phi}$. Lemma 2(ii) implies that $F_{B,\pi}$ is strictly decreasing, so this equation has at most one solution. Part (iii) gives existence and shows that the unique scalar solution uniquely determines $\mathbf{q}$ and $\mathbf{x}$ through (8). Hence the type-symmetric simultaneous equilibrium is unique. □

A.4 Proof of Lemma 1

The sequential objective is $W^{\text{seq}}(\mathbf{q}) = \alpha \sum_{\ell=1}^{d} m_\ell q_\ell^2 [1 + L(\mathbf{q}) k_\ell]$. Define $C(\mathbf{q}) := \sum_{j=1}^{d} m_j k_j q_j^2$. Then $W^{\text{seq}}(\mathbf{q}) = \alpha \sum_{j=1}^{d} m_j q_j^2 + \alpha L(\mathbf{q}) C(\mathbf{q})$.

Since $L(\mathbf{q}) = \frac{\phi \mu(\mathbf{q})}{1 - \phi \kappa(\mathbf{q})}$, differentiating with respect to q_ℓ gives $\frac{\partial L(\mathbf{q})}{\partial q_\ell} = \frac{\phi \pi_\ell [1 + L(\mathbf{q}) k_\ell]}{1 - \phi \kappa(\mathbf{q})}$. Also, $\frac{\partial C(\mathbf{q})}{\partial q_\ell} = 2 m_\ell k_\ell q_\ell$. Hence

$$\frac{\partial W^{\text{seq}}(\mathbf{q})}{\partial q_\ell} = 2\alpha m_\ell q_\ell [1 + L(\mathbf{q}) k_\ell] + \alpha C(\mathbf{q}) \frac{\partial L(\mathbf{q})}{\partial q_\ell}.$$

Substituting the derivative of $L(\mathbf{q})$ and dividing by m_ℓ gives

$$\frac{1}{m_\ell} \frac{\partial W^{\text{seq}}(\mathbf{q})}{\partial q_\ell} = 2\alpha q_\ell [1 + L(\mathbf{q}) k_\ell] + \alpha \frac{\phi C(\mathbf{q})}{1 - \phi \kappa(\mathbf{q})} [1 + L(\mathbf{q}) k_\ell] \frac{\pi_\ell}{m_\ell}.$$

Using $x_\ell(\mathbf{q}) = \alpha q_\ell [1 + L(\mathbf{q}) k_\ell]$, $r_\ell = \frac{\pi_\ell}{m_\ell}$, and $\eta(\mathbf{q}) := \frac{\phi C(\mathbf{q})}{1 - \phi \kappa(\mathbf{q})}$, this becomes

$$\frac{1}{m_\ell} \frac{\partial W^{\text{seq}}(\mathbf{q})}{\partial q_\ell} = 2 x_\ell(\mathbf{q}) + \alpha \eta(\mathbf{q}) [1 + L(\mathbf{q}) k_\ell] r_\ell.$$

This completes the proof. □

A.5 Proof of Proposition 2

Let $\mathbf{q}^{\text{sim}} := \mathbf{q}^{\text{sim}}(B)$, and write $T := T(B)$. Suppose there exist $a, b \in T$ such that $\mathcal{A}_a(B) > \mathcal{A}_b(B)$. Since both classes are partially targeted, $0 < q_a^{\text{sim}} < 1$ and $0 < q_b^{\text{sim}} < 1$.

Choose $\varepsilon > 0$ such that $q_a^{\text{sim}} - \frac{\varepsilon}{m_a} \geq 0$ and $q_b^{\text{sim}} + \frac{\varepsilon}{m_b} \leq 1$. Define $\mathbf{q}(\varepsilon)$ by

$$q_a(\varepsilon) = q_a^{\text{sim}} - \frac{\varepsilon}{m_a}, \qquad q_b(\varepsilon) = q_b^{\text{sim}} + \frac{\varepsilon}{m_b}, \qquad q_\ell(\varepsilon) = q_\ell^{\text{sim}} \quad \text{for } \ell \notin \{a, b\}.$$

Then $\mathbf{q}(\varepsilon) \in \mathcal{Q}(B)$. The perturbation is also budget neutral as it increases enforcement on class a by ε and reduces enforcement on class b by ε.

By Lemma 1, evaluated at $\mathbf{q}^{\text{sim}}$, we have

$$\frac{d}{d\varepsilon} W^{\text{seq}}(\mathbf{q}(\varepsilon))\Big|_{\varepsilon=0} = -\frac{1}{m_a}\frac{\partial W^{\text{seq}}(\mathbf{q}^{\text{sim}})}{\partial q_a} + \frac{1}{m_b}\frac{\partial W^{\text{seq}}(\mathbf{q}^{\text{sim}})}{\partial q_b}.$$

Let $L^{\text{sim}} := L(\mathbf{q}^{\text{sim}})$ and $\eta^{\text{sim}} := \eta(\mathbf{q}^{\text{sim}})$. Since $\frac{dq_a(\varepsilon)}{d\varepsilon} = -\frac{1}{m_a}$ and $\frac{dq_b(\varepsilon)}{d\varepsilon} = \frac{1}{m_b}$, Lemma 1 gives

$$\frac{d}{d\varepsilon} W^{\text{seq}}(\mathbf{q}(\varepsilon))\Big|_{\varepsilon=0} = 2x_b^{\text{sim}} - 2x_a^{\text{sim}} + \alpha\eta^{\text{sim}}\left([1 + L^{\text{sim}}k_b]r_b - [1 + L^{\text{sim}}k_a]r_a\right).$$

Because $a, b \in T(B)$, and both classes are partially targeted, the planner's optimality conditions in the simultaneous problem therefore imply $x_a^{\text{sim}} = x_b^{\text{sim}}$. Hence

$$\frac{d}{d\varepsilon} W^{\text{seq}}(\mathbf{q}(\varepsilon))\Big|_{\varepsilon=0} = -\alpha\eta^{\text{sim}}\big[\mathcal{A}_a(B) - \mathcal{A}_b(B)\big],$$

given the definition of $\mathcal{A}_\ell(B)$. Moreover, $\eta^{\text{sim}} > 0$: by Assumption 2, $\phi > 0$ and $1 - \phi\kappa(\mathbf{q}^{\text{sim}}) > 0$, while $C(\mathbf{q}^{\text{sim}}) > 0$, because $n - B > 0$ implies that at least one class has positive survival and $k_\ell > 0$ for every ℓ. Therefore $\mathcal{A}_a(B) > \mathcal{A}_b(B)$ implies $\frac{d}{d\varepsilon} W^{\text{seq}}(\mathbf{q}(\varepsilon))\big|_{\varepsilon=0} < 0$. So for all sufficiently small $\varepsilon > 0$, $W^{\text{seq}}(\mathbf{q}(\varepsilon)) < W^{\text{seq}}(\mathbf{q}^{\text{sim}})$. Hence $\mathbf{q}^{\text{sim}}(B)$ is not sequentially optimal. Since $\mathbf{q}^{\text{seq}}(B)$ minimises W^{seq}, we have $W^{\text{seq}}(\mathbf{q}^{\text{seq}}(B)) < W^{\text{seq}}(\mathbf{q}^{\text{sim}}(B))$. □

A.6 Proof of Corollary 1

Take any $i < j$ in the simultaneous targeted block $T(B)$. Then $k_i > k_j$, and by Assumption 3, $r_i \geq r_j$. Also, $r_j > 0$, because $\pi_j > 0$ and $m_j > 0$. Since $L^{\text{sim}}(B) > 0$, it follows that $[1 + L^{\text{sim}}(B)k_i]r_i > [1 + L^{\text{sim}}(B)k_j]r_j$. Therefore $\mathcal{A}_i(B) > \mathcal{A}_j(B)$.

If $|T(B)| \geq 2$, the simultaneous targeted block contains two classes with different amplification scores. Proposition 2 therefore implies that the simultaneous policy is not optimal for the sequential problem: $W^{\text{seq}}(\mathbf{q}^{\text{seq}}(B)) < W^{\text{seq}}(\mathbf{q}^{\text{sim}}(B))$. This completes the proof. □

A.7 Proofs for Subsection 4.4

Proof of Proposition 3. Assumption 2 implies $1-\phi\bar{k}>0$, and hence $1+\phi[k_1-\bar{k}+\pi_1(k_1-k_2)]>\phi(k_1-k_2)>0$. The definition of $B_c(\phi)$ gives $0<B_c(\phi)<m_1<n$.

By Proposition 1, any simultaneous equilibrium targeting a single class targets class 1. Lemma 2(iii) implies $q_1>0$. The budget constraint gives $B<m_1$ and the survival profile $\mathbf{q}^0=(s,1,\ldots,1)$, where $s=1-B/m_1$. For $0<B<m_1$, equation (2) implies that class 2 has the highest effort among untargeted classes. The planner's optimality conditions in Appendix A.1 hold at $\mathbf{q}^0$ if and only if $x_1(\mathbf{q}^0)\geq x_2(\mathbf{q}^0)$. Substituting $\mathbf{q}^0$ into (2) gives

$$\frac{x_1(\mathbf{q}^0)-x_2(\mathbf{q}^0)}{\alpha}=\frac{\phi(k_1-k_2)-\frac{B}{m_1}\left\{1+\phi\left[k_1-\bar{k}+\pi_1(k_1-k_2)\right]\right\}}{1-\phi\kappa(\mathbf{q}^0)}.$$

By Assumption 2, $1-\phi\kappa(\mathbf{q}^0)\geq 1-\phi\bar{k}>0$. Thus $x_1(\mathbf{q}^0)\geq x_2(\mathbf{q}^0)$ if and only if $B\leq B_c(\phi)$. The uniqueness result in Proposition 1 gives $T(B)=\{1\}$ if and only if $0<B\leq B_c(\phi)$. At $B=B_c(\phi)$, class 2 remains untargeted, with $q_2^0=1$, although $x_1(\mathbf{q}^0)=x_2(\mathbf{q}^0)$.

Fix $0<B\leq B_c(\phi)$ and $\mathbf{q}\in\mathcal{Q}(B)$. Let $D_j=m_j(1-q_j)\geq 0$ for $j\geq 2$. The budget constraint implies $m_1(q_1-s)=\sum_{j=2}^{d}D_j$. With $r_\ell=\pi_\ell/m_\ell$, Definition 2 gives

$$\mu(\mathbf{q})-\mu(\mathbf{q}^0)=\sum_{j=2}^{d}D_j(r_1-r_j)\geq 0,$$

$$\kappa(\mathbf{q})-\kappa(\mathbf{q}^0)=\sum_{j=2}^{d}D_j(k_1r_1-k_jr_j)\geq 0,$$

where both inequalities follow from Assumption 3.

The preceding inequalities imply $L(\mathbf{q})\geq L(\mathbf{q}^0)>0$ by Definition 2 and Assumption 2. Holding $L(\mathbf{q}^0)$ fixed, define $F(\mathbf{q})=\alpha\sum_{\ell=1}^{d}m_\ell q_\ell^2[1+L(\mathbf{q}^0)k_\ell]$. Equation (2) gives $\partial F(\mathbf{q}^0)/\partial q_\ell=2m_\ell x_\ell(\mathbf{q}^0)$ for every ℓ. The planner's optimality conditions in Appendix A.1 imply $\nabla F(\mathbf{q}^0)\cdot(\mathbf{q}-\mathbf{q}^0)\geq 0$ for every $\mathbf{q}\in\mathcal{Q}(B)$. Since F is strictly convex, $\mathbf{q}^0$ is its unique minimizer on $\mathcal{Q}(B)$. Every $\mathbf{q}\in\mathcal{Q}(B)\setminus\{\mathbf{q}^0\}$ satisfies

$$W^{\text{seq}}(\mathbf{q})\geq F(\mathbf{q})>F(\mathbf{q}^0)=W^{\text{seq}}(\mathbf{q}^0).$$

The profile $\mathbf{q}^0$ is then the unique global sequential optimum, and $G(B,\phi)=0$ for $0<B\leq B_c(\phi)$. Also for $B>B_c(\phi)$, the simultaneous equilibrium targets at least two classes and corollary 1 obtains $G(B,\phi)>0$. ■

For the corollaries, extend the model to $\phi=0$. Equation (2) gives $x_\ell(\mathbf{q};0)=\alpha q_\ell$. The simultaneous planner's optimality conditions in Appendix A.1 coincide with those for minimizing the sequential objective $\alpha\sum_\ell m_\ell q_\ell^2$ over $\mathcal{Q}(B)$. This strictly convex problem has the unique solution $q_\ell=(n-B)/n$ for every ℓ, so $G(B,0)=0$.

For $0<B<n$ and $0\leq\phi<1/\bar{k}$, the feasible sets $\mathcal{Q}(B)$ are compact and vary continuously with B, and the effort formula in (2) is jointly continuous. Along any convergent sequence of parameters, passing to limits in the planner's optimality conditions shows that every limit point of simultaneous profiles is an equilibrium at the limiting parameters. Uniqueness, established above at $\phi=0$ and in Proposition 1 for $\phi>0$, implies joint continuity of the simultaneous profile and W^{sim}. Continuity of the objective and the feasible correspondence also gives joint continuity of $W^{\text{seq}}_{\min}$. Hence G is jointly continuous on this domain.

Proof of Corollary 2. Fix $0 < \phi < 1/\bar{k}$. The budget constraint gives $0 \le q_\ell \le (n-B)/m_\ell$, so feasible profiles converge uniformly to $\mathbf{0}$ as $B \to n^-$. Since W^{seq} is continuous and $W^{\text{seq}}(\mathbf{0}) = 0$, we have $W^{\text{sim}}(B,\phi) \to 0$. The bound $0 \le G(B,\phi) \le W^{\text{sim}}(B,\phi)$ implies $G(B,\phi) \to 0$.

Proposition 3 gives $G(B,\phi) = 0$ for $0 < B \le B_c(\phi)$ and $G(B,\phi) > 0$ for $B_c(\phi) < B < n$. Extend $G(\cdot,\phi)$ to $[0,n]$ by setting $G(0,\phi) = G(n,\phi) = 0$. This continuous extension attains a strictly positive maximum, and every maximizer lies in $(B_c(\phi), n)$. Thus $G(\cdot,\phi)$ is nonmonotone. ■

Proof of Corollary 3. Fix $0 < B < \frac{m_1(k_1-k_2)}{k_1+\pi_1(k_1-k_2)}$. The budget restriction gives

$$m_1(k_1-k_2) - B\left[k_1 - \bar{k} + \pi_1(k_1-k_2)\right] > B\bar{k} > 0.$$

The cutoff ϕ_c is well defined, with $0 < \phi_c < 1/\bar{k}$. For $0 < \phi < 1/\bar{k}$, the condition $B \le B_c(\phi)$ is equivalent to $\phi \ge \phi_c$. By Proposition 3, $G(B,\phi) > 0$ for $0 < \phi < \phi_c$, and $G(B,\phi) = 0$ for $\phi_c \le \phi < 1/\bar{k}$.

Continuity gives $G(B,\phi) \to G(B,0) = 0$ as $\phi \to 0^+$. On $[0,\phi_c]$, the function $G(B,\cdot)$ is continuous, positive in the interior, and zero at both endpoints. It attains a strictly positive maximum at some $\phi^* \in (0,\phi_c)$. Since $G(B,\phi) = 0$ for $\phi_c \le \phi < 1/\bar{k}$, this maximum is global on $(0,1/\bar{k})$, and $G(B,\cdot)$ is nonmonotone.

For fixed $(\mathbf{k},\mathbf{m},\boldsymbol{\pi})$,

$$B_c'(\phi) = \frac{m_1(k_1-k_2)}{\left\{1+\phi\left[k_1-\bar{k}+\pi_1(k_1-k_2)\right]\right\}^2} > 0.$$

The threshold is strictly increasing and satisfies $\lim_{\phi\to(1/\bar{k})^-} B_c(\phi) = \frac{m_1(k_1-k_2)}{k_1+\pi_1(k_1-k_2)}$. If B is at least this limit, $B > B_c(\phi)$ for every $0 < \phi < 1/\bar{k}$. Proposition 3 gives $G(B,\phi) > 0$ throughout this interval. The strict budget restriction is necessary for the gain to vanish at some $\phi \in (0,1/\bar{k})$. ■

A.8 Proof of Proposition 4

Write the budget constraint as $\sum_{\ell=1}^d m_\ell q_\ell = n - B$, and let $\mathbf{q}^{\text{seq}}$ be a sequential optimum. Since $\mathcal{Q}(B)$ is convex and W^{seq} is continuously differentiable, the first-order necessary condition is $\nabla W^{\text{seq}}(\mathbf{q}^{\text{seq}}) \cdot (\mathbf{q} - \mathbf{q}^{\text{seq}}) \ge 0$ for every $\mathbf{q} \in \mathcal{Q}(B)$. The proof uses only this necessary condition and does not require convexity of W^{seq}.

Fix a sequential optimum $\mathbf{q}^{\text{seq}}(B)$, and let λ^{seq} be the multiplier on $\sum_{\ell=1}^d m_\ell q_\ell = n - B$. For each class ℓ, define the per-capita marginal cost of survival as $M_\ell^{\text{seq}} := \frac{1}{m_\ell}\frac{\partial W^{\text{seq}}(\mathbf{q}^{\text{seq}}(B))}{\partial q_\ell}$.

The first-order condition implies, for each ℓ,

$$\begin{aligned} M_\ell^{\text{seq}} &\ge \lambda^{\text{seq}} \quad \text{if } q_\ell^{\text{seq}} = 0, \\ M_\ell^{\text{seq}} &= \lambda^{\text{seq}} \quad \text{if } 0 < q_\ell^{\text{seq}} < 1, \\ M_\ell^{\text{seq}} &\le \lambda^{\text{seq}} \quad \text{if } q_\ell^{\text{seq}} = 1. \end{aligned}$$

Write $L^{\text{seq}} := L(\mathbf{q}^{\text{seq}}(B))$ and $\eta^{\text{seq}} := \eta(\mathbf{q}^{\text{seq}}(B))$. Lemma 1 gives $M_\ell^{\text{seq}} = \alpha[1+L^{\text{seq}}k_\ell][2q_\ell^{\text{seq}} + \eta^{\text{seq}}r_\ell]$ for every ℓ. Since $B < n$, the budget constraint implies that at least one class has positive survival. For every such class, the marginal-cost formula gives $M_\ell^{\text{seq}} > 0$. Choose a class ℓ with

$q_\ell^{\text{seq}}(B) > 0$. The first-order conditions give $\lambda^{\text{seq}} \geq M_\ell^{\text{seq}} > 0$. The value of $q_\ell^{\text{seq}}(B)$ that would satisfy $M_\ell^{\text{seq}} = \lambda^{\text{seq}}$ is therefore

$$z_\ell := \frac{1}{2}\left[\frac{\lambda^{\text{seq}}}{\alpha[1 + L^{\text{seq}} k_\ell]} - \eta^{\text{seq}} r_\ell\right].$$

The first-order conditions then give $q_\ell^{\text{seq}}(B) = \min\{1, \max\{0, z_\ell\}\}$.

If $0 < q_\ell^{\text{seq}}(B) < 1$, the condition $M_\ell^{\text{seq}} = \lambda^{\text{seq}}$ gives $q_\ell^{\text{seq}}(B) = z_\ell$. At $q_\ell^{\text{seq}}(B) = 0$, $M_\ell^{\text{seq}} \geq \lambda^{\text{seq}}$ implies $z_\ell \leq 0$; at $q_\ell^{\text{seq}}(B) = 1$, $M_\ell^{\text{seq}} \leq \lambda^{\text{seq}}$ implies $z_\ell \geq 1$.

Fix $i < j$, so $k_i > k_j$. Assumption 3 gives $r_i \geq r_j$. Since $L^{\text{seq}} > 0$, $\lambda^{\text{seq}} > 0$, and $\eta^{\text{seq}} \geq 0$,

$$\frac{\lambda^{\text{seq}}}{\alpha[1 + L^{\text{seq}} k_i]} < \frac{\lambda^{\text{seq}}}{\alpha[1 + L^{\text{seq}} k_j]}, \qquad \eta^{\text{seq}} r_i \geq \eta^{\text{seq}} r_j.$$

These inequalities imply $z_i < z_j$. The map $z \mapsto \min\{1, \max\{0, z\}\}$ is weakly increasing, so $q_i^{\text{seq}}(B) \leq q_j^{\text{seq}}(B)$.

Since $q_\ell^{\text{seq}}(B)$ is weakly increasing in ℓ, fully removed classes form an initial block and untargeted classes form a terminal block. Any partially targeted classes lie between these blocks. Either boundary block may be empty. There exist cutoffs $0 \leq h^{\text{seq}} \leq \tau^{\text{seq}} \leq d$ such that

$$\begin{aligned} q_\ell^{\text{seq}}(B) &= 0 \quad \text{for } \ell \leq h^{\text{seq}}, \\ 0 < q_\ell^{\text{seq}}(B) &< 1 \quad \text{for } h^{\text{seq}} < \ell \leq \tau^{\text{seq}}, \\ q_\ell^{\text{seq}}(B) &= 1 \quad \text{for } \ell > \tau^{\text{seq}}. \end{aligned}$$

For partially targeted classes $i < j$, $q_i^{\text{seq}}(B) = z_i < z_j = q_j^{\text{seq}}(B)$. Survival is strictly increasing in the class index within this block. □

A.9 Proof of Proposition 5

Since $i, j \in T^s$, Proposition 7 implies $0 < q_i^s, q_j^s < 1$. Choose $\varepsilon > 0$ small enough that $q_i^s - \frac{\varepsilon}{m_i} \geq 0$ and $q_j^s + \frac{\varepsilon}{m_j} \leq 1$. Define $\mathbf{q}(\varepsilon) := \mathbf{q}^s - \frac{\varepsilon}{m_i}\mathbf{e}_i + \frac{\varepsilon}{m_j}\mathbf{e}_j$. This perturbation preserves the budget constraint: it increases enforcement on class i by ε and reduces enforcement on class j by ε. Hence $\mathbf{q}(\varepsilon) \in \mathcal{Q}(B)$, and the perturbation shifts enforcement from class j toward class i.

By Lemma 4,

$$\frac{d}{d\varepsilon}\widetilde{W}(\mathbf{q}(\varepsilon))\bigg|_{\varepsilon=0} = -\frac{1}{m_i}\frac{\partial \widetilde{W}(\mathbf{q}^s)}{\partial q_i} + \frac{1}{m_j}\frac{\partial \widetilde{W}(\mathbf{q}^s)}{\partial q_j}.$$

Let $\widetilde{\eta}^s := \widetilde{\eta}(\mathbf{q}^s)$. Since i and j are partially targeted in the simultaneous equilibrium, their efforts are equalised. Write the common effort as $x^s > 0$. Then

$$\frac{d}{d\varepsilon}\widetilde{W}(\mathbf{q}(\varepsilon))\bigg|_{\varepsilon=0} = -\left[2x^s + 2\widetilde{\eta}^s x^s r_i\right] + \left[2x^s + 2\widetilde{\eta}^s x^s r_j\right] = -2\widetilde{\eta}^s x^s (r_i - r_j).$$

Moreover, $\widetilde{\eta}^s > 0$, because $\phi > 0$, $1 - \phi\widetilde{\kappa}(\mathbf{q}^s) > 0$, and $\widetilde{C}(\mathbf{q}^s) > 0$. Hence the derivative is strictly negative whenever $r_i > r_j$. Therefore, for all sufficiently small $\varepsilon > 0$, $\widetilde{W}(\mathbf{q}(\varepsilon)) < \widetilde{W}(\mathbf{q}^s)$. Thus $\mathbf{q}^s$ is not sequentially optimal. Every sequential optimum yields strictly less surviving activity than $\mathbf{q}^s$ does. □

A.10 Proof of Proposition 6

The proof uses the scalar characterization in Lemma 2.

Fix $B \in (0, n)$ and a conjectured spillover index $L > 0$. The fixed-effort planner's threshold rule gives $q_\ell(L; B) = \min\{1, \frac{z_B(L)}{1+Lk_\ell}\}$. Hence induced effort is $x_\ell(L; B) = \alpha q_\ell(L; B)(1 + Lk_\ell) = \alpha \min\{z_B(L), 1 + Lk_\ell\}$. Therefore, at the simultaneous equilibrium,

$$x^*_\ell(B, \phi, \pi) = \alpha \min\{z_B(L^*(B, \phi, \pi)), 1 + L^*(B, \phi, \pi)k_\ell\}. \tag{9}$$

Using $w_B(L) = \frac{z_B(L)}{L}$ and $h_\ell(L) = \frac{1}{L} + k_\ell$, the fixed-point condition for the spillover index is equivalent to

$$F_{B,\pi}(L^*(B, \phi, \pi)) = \frac{1}{\phi}. \tag{10}$$

Thus $L^*(B, \phi, \pi)$ is the unique solution to $F_{B,\pi}(L) = \frac{1}{\phi}$. Existence and uniqueness follow directly from Lemma 2(ii)–(iii).

We now prove the three claims in Proposition 6.

Part (i). Let $B' > B$, holding (ϕ, π) fixed. Write $L_B := L^*(B, \phi, \pi)$ and $L_{B'} := L^*(B', \phi, \pi)$. By (10), $F_{B,\pi}(L_B) = \frac{1}{\phi}$ and $F_{B',\pi}(L_{B'}) = \frac{1}{\phi}$. Lemma 2(iv) implies $F_{B',\pi}(L_B) < F_{B,\pi}(L_B) = \frac{1}{\phi}$. Since $F_{B',\pi}$ is strictly decreasing in L by Lemma 2(ii), the unique solution to $F_{B',\pi}(L) = \frac{1}{\phi}$ satisfies $L_{B'} < L_B$. Therefore $L^*(B', \phi, \pi) < L^*(B, \phi, \pi)$.

We next prove the effort comparison. Since $L_{B'} < L_B$, Lemma 2(i) gives $z_B(L_{B'}) < z_B(L_B)$. Since $B' > B$, Lemma 2(i) gives $z_{B'}(L_{B'}) < z_B(L_{B'})$. Hence $z_{B'}(L_{B'}) < z_B(L_B)$. Also, $1 + L_{B'}k_\ell < 1 + L_Bk_\ell$ for every ℓ. By (9), both arguments of the minimum are strictly lower under $(B', L_{B'})$ than under (B, L_B). Therefore $x^*_\ell(B', \phi, \pi) < x^*_\ell(B, \phi, \pi)$ for every ℓ.

It remains to prove the comparison of targeting cutoffs in part (i). Class ℓ is targeted if and only if $q_\ell(L; B) < 1$. Since $q_\ell(L; B) = \min\{1, \frac{z_B(L)}{1+Lk_\ell}\}$, this is equivalent to $z_B(L) < 1 + Lk_\ell$, or $c_B(L) < k_\ell$. Since $B' > B$, Lemma 2(i) gives $c_{B'}(L) < c_B(L)$ for every fixed L. Since $L_{B'} < L_B$, and $c_{B'}(L)$ is weakly increasing in L by Lemma 2(i), $c_{B'}(L_{B'}) \le c_{B'}(L_B) < c_B(L_B)$. Thus $c_B(L_B) < k_\ell$ implies $c_{B'}(L_{B'}) < k_\ell$. Therefore every class targeted under B remains targeted under B'. Since targeted classes form an initial block by Proposition 1, $\tau^*(B', \phi, \pi) \ge \tau^*(B, \phi, \pi)$.

Part (ii). Fix (B, π), and let $\phi'' > \phi'$, with both values satisfying Assumption 2. Write $L' := L^*(B, \phi', \pi)$ and $L'' := L^*(B, \phi'', \pi)$. By (10), $F_{B,\pi}(L') = \frac{1}{\phi'}$ and $F_{B,\pi}(L'') = \frac{1}{\phi''}$. Since $\frac{1}{\phi''} < \frac{1}{\phi'}$, and $F_{B,\pi}$ is strictly decreasing by Lemma 2(ii), the unique solution under ϕ'' satisfies $L'' > L'$. Hence $L^*(B, \phi'', \pi) > L^*(B, \phi', \pi)$.

We next prove the effort comparison. Since $L'' > L'$, Lemma 2(i) gives $z_B(L'') > z_B(L')$. Also, $1 + L''k_\ell > 1 + L'k_\ell$ for every ℓ. By (9), both arguments of the minimum are strictly higher under (B, L'') than under (B, L'). Therefore $x^*_\ell(B, \phi'', \pi) > x^*_\ell(B, \phi', \pi)$ for every ℓ.

It remains to prove the comparison of targeting cutoffs in part (ii). Since $q_\ell(L; B) = \min\{1, \frac{z_B(L)}{1+Lk_\ell}\}$, class ℓ is targeted if and only if $z_B(L) < 1 + Lk_\ell$, or equivalently $c_B(L) < k_\ell$. Since $L'' > L'$, Lemma 2(i) gives $c_B(L'') \ge c_B(L')$. Thus any class targeted under ϕ'' is also targeted under ϕ'. Since targeted classes form an initial block by Proposition 1, $\tau^*(B, \phi'', \pi) \le \tau^*(B, \phi', \pi)$.

Part (iii). Fix (B, ϕ). Suppose the distribution of neighbor degree under π' first-order stochastically dominates that under π, so that $\sum_{\ell=1}^s \pi'_\ell \ge \sum_{\ell=1}^s \pi_\ell$ for every $s = 1, \ldots, d-1$. Write $L_\pi := L^*(B, \phi, \pi)$ and $L_{\pi'} := L^*(B, \phi, \pi')$. By (10), $F_{B,\pi}(L_\pi) = \frac{1}{\phi}$ and $F_{B,\pi'}(L_{\pi'}) = \frac{1}{\phi}$.

Lemma 2(v) implies $F_{B,\pi'}(L_\pi) \geq F_{B,\pi}(L_\pi) = \frac{1}{\phi}$. Since $F_{B,\pi'}$ is strictly decreasing in L by Lemma 2(ii), its unique solution to $F_{B,\pi'}(L) = \frac{1}{\phi}$ satisfies $L_{\pi'} \geq L_\pi$. Therefore $L^*(B,\phi,\pi') \geq L^*(B,\phi,\pi)$.

We next prove the effort comparison. Since $L_{\pi'} \geq L_\pi$, Lemma 2(i) gives $z_B(L_{\pi'}) \geq z_B(L_\pi)$. Also, $1 + L_{\pi'}k_\ell \geq 1 + L_\pi k_\ell$ for every ℓ. By (9), both arguments of the minimum are weakly higher under π' than under π. Therefore $x^*_\ell(B,\phi,\pi') \geq x^*_\ell(B,\phi,\pi)$ for every ℓ.

It remains to prove the comparison of targeting cutoffs in part (iii). Since $q_\ell(L;B) = \min\{1, \frac{z_B(L)}{1+Lk_\ell}\}$, class ℓ is targeted if and only if $z_B(L) < 1+Lk_\ell$, or equivalently $c_B(L) < k_\ell$. Since $L_{\pi'} \geq L_\pi$, Lemma 2(i) gives $c_B(L_{\pi'}) \geq c_B(L_\pi)$. Thus any class targeted under π' is also targeted under π. Since targeted classes form an initial block by Proposition 1, $\tau^*(B,\phi,\pi') \leq \tau^*(B,\phi,\pi)$. □

B Supplementary results

B.1 A surviving-neighbor spillover variant

In the baseline model, enforcement changes spillovers through players' effort choices. In this variant, it also removes the neighbor activity entering the complementarity term. A type-ℓ player's payoff is

$$\widetilde{U}_\ell(\mathbf{x},\mathbf{q}) = \alpha q_\ell x_\ell - \frac{1}{2}x_\ell^2 + \phi k_\ell q_\ell x_\ell \bar{y}, \qquad \bar{y} := \sum_{j=1}^{d} \pi_j q_j x_j.$$

Here $\bar{y}$ is expected surviving-neighbor effort. Relative to (1), the return from neighbors now depends on their survival as well as on their effort. Enforcement therefore lowers both a player's own return to effort and the neighbor activity generating complementarities. The following lemma gives the effort response for a fixed survival profile.

Lemma 3 (Reduced-form effort under surviving-neighbor spillovers) *Under Assumptions 1 and 2, for every $\mathbf{q} \in \mathcal{Q}(B)$, there is a unique type-symmetric effort profile. It is given by $x_\ell(\mathbf{q}) = \alpha q_\ell[1 + \widetilde{L}(\mathbf{q})k_\ell]$, where*

$$\widetilde{L}(\mathbf{q}) = \frac{\phi \sum_{j=1}^{d} \pi_j q_j^2}{1 - \phi \sum_{j=1}^{d} \pi_j k_j q_j^2}.$$

Moreover, $\widetilde{L}(\mathbf{q}) > 0$ for every $\mathbf{q} \in \mathcal{Q}(B)$.

Proof. Fix $\mathbf{q} \in \mathcal{Q}(B)$. Taking $\bar{y}$ as given, a type-ℓ player solves

$$\max_{x_\ell \geq 0} \left\{ \alpha q_\ell x_\ell - \frac{1}{2}x_\ell^2 + \phi k_\ell q_\ell x_\ell \bar{y} \right\}.$$

The objective is strictly concave in x_ℓ. Hence the unique best response is $x_\ell = \alpha q_\ell + \phi k_\ell q_\ell \bar{y}$. In a type-symmetric profile, $\bar{y} = \sum_{\ell=1}^{d} \pi_\ell q_\ell x_\ell$. Substituting the best responses into this consistency condition gives $\bar{y} = \frac{\alpha \sum_{\ell=1}^{d} \pi_\ell q_\ell^2}{1 - \phi \sum_{\ell=1}^{d} \pi_\ell k_\ell q_\ell^2}$. Assumption 2 implies $1 - \phi \sum_{\ell=1}^{d} \pi_\ell k_\ell q_\ell^2 \geq 1 - \phi \sum_{\ell=1}^{d} \pi_\ell k_\ell > 0$. Substituting this expression for $\bar{y}$ into the best response gives the stated effort equation.

Finally, since $B < n$, some type has positive survival. Since $\pi_\ell > 0$ for every ℓ, the numerator of $\widetilde{L}(\mathbf{q})$ is positive. Since $\phi > 0$, it follows that $\widetilde{L}(\mathbf{q}) > 0$. ■

The planner still takes effort as fixed in the simultaneous game and assigns enforcement to classes with the highest effort. The next proposition establishes that cutoff targeting, positive survival in every class, and effort equalization continue to hold.

Proposition 7 (Simultaneous targeting under surviving-neighbor spillovers) *Under Assumptions 1 and 2, a type-symmetric simultaneous equilibrium exists. In every type-symmetric simultaneous equilibrium:*

(i) targeting has cutoff form: if $q_s < 1$, then $q_r < 1$ for every $r < s$;

(ii) no class is fully removed: $q_\ell > 0$ for every ℓ;

(iii) effort is equalised across all partially targeted classes.

Proof. Let $\mathcal{Q}(B) := \{\mathbf{q} \in [0,1]^d : \sum_{\ell=1}^d m_\ell(1 - q_\ell) = B\}$. Since $B \in (0, n)$, this set is nonempty, compact, and convex. By Lemma 3, the induced effort profile $\mathbf{x}(\mathbf{q})$ is continuous on $\mathcal{Q}(B)$. Define the planner's best-response correspondence by

$$G(\mathbf{q}) := \arg\min_{\mathbf{q}' \in \mathcal{Q}(B)} \sum_{\ell=1}^{d} m_\ell q'_\ell x_\ell(\mathbf{q}).$$

The objective is continuous in $(\mathbf{q}', \mathbf{q})$ and linear in $\mathbf{q}'$. Hence G has nonempty, compact, convex values and is upper hemicontinuous. Kakutani's fixed-point theorem therefore gives $\mathbf{q}^* \in \mathcal{Q}(B)$ such that $\mathbf{q}^* \in G(\mathbf{q}^*)$. Let $\mathbf{x}^* := \mathbf{x}(\mathbf{q}^*)$. Then $(\mathbf{q}^*, \mathbf{x}^*)$ is a type-symmetric simultaneous equilibrium.

We next prove the cutoff property. Suppose, to the contrary, that there exist $r < s$ such that $q_r^* = 1$ and $q_s^* < 1$. Since $k_r > k_s$ and $q_r^* > q_s^*$, Lemma 3 implies $x_r^* > x_s^*$. Choose $\varepsilon > 0$ small enough that $q_r^* - \frac{\varepsilon}{m_r} \geq 0$ and $q_s^* + \frac{\varepsilon}{m_s} \leq 1$. Reduce q_r^* by $\frac{\varepsilon}{m_r}$, increase q_s^* by $\frac{\varepsilon}{m_s}$, and leave all other survival probabilities unchanged. This deviation is feasible and, holding effort fixed, changes the planner's objective by $\varepsilon(x_s^* - x_r^*) < 0$, contradicting the planner's optimality. Hence no lower-degree class is targeted while a higher-degree class remains untargeted.

Now suppose $q_r^* = 0$ for some r. Then $x_r^* = 0$. Since $B < n$, some class s has $q_s^* > 0$ and hence $x_s^* > 0$. Choose $\varepsilon > 0$ small enough that $\frac{\varepsilon}{m_r} \leq 1$ and $\frac{\varepsilon}{m_s} \leq q_s^*$. Increase q_r^* by $\frac{\varepsilon}{m_r}$, reduce q_s^* by $\frac{\varepsilon}{m_s}$, and leave all other survival probabilities unchanged. This deviation is feasible and, holding effort fixed, changes the planner's objective by $\varepsilon(x_r^* - x_s^*) < 0$, again contradicting the planner's optimality. Therefore $q_\ell^* > 0$ for every ℓ.

Finally, fix a simultaneous equilibrium. Given the effort profile, the planner solves the linear program $\min_{\mathbf{q} \in \mathcal{Q}(B)} \sum_{\ell=1}^d m_\ell q_\ell x_\ell$. The first-order conditions are necessary and sufficient for this problem. Hence there exists a Lagrange multiplier λ such that $x_\ell = \lambda$ whenever $0 < q_\ell < 1$. Since no class is fully removed, every targeted class is interior. Therefore effort is equalised across all targeted classes. ■

Proposition 7 gives existence and the structure of simultaneous equilibria. For uniqueness, we additionally impose Assumption 3. The budget and the planner's optimality conditions reduce equilibrium to a scalar consistency condition. Unlike in Proposition 1, this condition involves q_ℓ^2. At the final step of the proof, the ordering of r_ℓ makes the consistency map strictly monotone. Without that ordering, the baseline uniqueness argument need not apply.

Proposition 8 (Uniqueness under surviving-neighbor spillovers) *Suppose Assumptions 1, 2, and 3 hold. Then the surviving-neighbour simultaneous game has a unique type-symmetric equilibrium.*

Proof.

Fix a candidate surviving-neighbour spillover index $L > 0$. By simultaneous optimality and no complete removal, any equilibrium with spillover index L has the form $q_\ell = \min\{1, \frac{z}{1+Lk_\ell}\}$ for some $z > 0$. Let $w := \frac{z}{L}$ and $a_\ell(L) := \frac{L}{1+Lk_\ell}$. Then $q_\ell = \min\{1, wa_\ell(L)\}$.

For each fixed $L > 0$, the budget equation $\sum_{\ell=1}^{d} m_\ell \min\{1, wa_\ell(L)\} = n - B$ has a unique solution in w. The left-hand side is continuous, equals zero at $w = 0$, reaches n for large w, and is strictly increasing whenever it is below n. Since $n - B \in (0, n)$, existence and uniqueness follow. Denote the solution by $w_B(L)$, and define $q_\ell(L) := \min\{1, w_B(L)a_\ell(L)\}$. Thus any equilibrium with spillover index L must induce this survival profile.

Surviving-neighbour spillover consistency requires $L = \frac{\phi \sum_{\ell=1}^{d} \pi_\ell q_\ell(L)^2}{1-\phi \sum_{\ell=1}^{d} \pi_\ell k_\ell q_\ell(L)^2}$. Since $\frac{1}{a_\ell(L)} = \frac{1}{L} + k_\ell$, this condition is equivalent to $1 = \Phi(L)$, where $\Phi(L) := \phi \sum_{\ell=1}^{d} \pi_\ell \frac{q_\ell(L)^2}{a_\ell(L)}$. It is therefore enough to show that Φ is strictly decreasing on $(0, \infty)$.

We first show that $w_B(L)$ is continuous in L. Let $G(L, w) := \sum_{\ell=1}^{d} m_\ell \min\{1, wa_\ell(L)\}$. This function is continuous in (L, w). Fix $L > 0$, and take any sequence $L_s \to L$. Write $w_s := w_B(L_s)$. For all large s, the values L_s stay in a compact subset of $(0, \infty)$. Hence $a_\ell(L_s)$ is uniformly bounded above and away from zero for every ℓ. Since $G(L_s, w_s) = n - B \in (0, n)$, the sequence $\{w_s\}$ is bounded.

Now take any convergent subsequence $w_{s_r} \to \bar{w}$. By continuity, $G(L, \bar{w}) = n - B$. The budget equation has a unique solution at L, so $\bar{w} = w_B(L)$. Thus every convergent subsequence of $\{w_s\}$ converges to $w_B(L)$, which implies $w_B(L_s) \to w_B(L)$. Therefore w_B, each $q_\ell(L)$, and $\Phi(L)$ are continuous.

Next define $z_B(L) := Lw_B(L)$ and $c_B(L) := \frac{z_B(L)-1}{L}$. Class ℓ is targeted if and only if $q_\ell(L) < 1$. Equivalently, $z_B(L) < 1 + Lk_\ell$, or $c_B(L) < k_\ell$. We claim that $c_B(L)$ is weakly increasing in L.

To see this, write $z = 1 + Lc$, and define $S(L, c) := \sum_{\ell=1}^{d} m_\ell \min\{1, \frac{1+Lc}{1+Lk_\ell}\}$. At the budget solution, $S(L, c_B(L)) = n - B$. For fixed c, each term of $S(L, c)$ is weakly decreasing in L: if $k_\ell > c$, then $\frac{1+Lc}{1+Lk_\ell}$ is strictly decreasing in L; if $k_\ell \leq c$, the capped term equals one. For fixed L, $S(L, c)$ is strictly increasing in c whenever its value is below n.

Now take $L_2 > L_1$. If $c_B(L_2) < c_B(L_1)$, then

$$S(L_2, c_B(L_2)) < S(L_2, c_B(L_1)) \leq S(L_1, c_B(L_1)) = n - B,$$

which contradicts $S(L_2, c_B(L_2)) = n - B$. Therefore $c_B(L_2) \geq c_B(L_1)$, so $c_B(L)$ is weakly increasing in L.

It follows that the targeted set $T(L) := \{\ell : k_\ell > c_B(L)\}$ is weakly decreasing in L. Since there are finitely many degree classes and c_B is continuous and weakly increasing, for any $0 < L_1 < L_2 < \infty$, the interval $[L_1, L_2]$ can be partitioned into finitely many subintervals on which $T(L)$ is constant in the interior. Changes in $T(L)$ can occur only when $c_B(L) = k_\ell$ for some ℓ.

Now consider an interval on which the targeted set is constant. Write this set as $T = \{1, \ldots, \tau\}$ and its complement as $U = \{\tau + 1, \ldots, d\}$. On this interval, $q_\ell(L) = w_B(L)a_\ell(L)$ for $\ell \in T$, and $q_\ell(L) = 1$ for $\ell \in U$. The budget equation therefore implies $w_B(L) = \frac{R_T}{A_T(L)}$, where

$R_T := n - B - \sum_{\ell \in U} m_\ell$ and $A_T(L) := \sum_{\ell \in T} m_\ell a_\ell(L)$. The set T is nonempty because $B > 0$. Moreover, $R_T > 0$, since all targeted classes have positive survival.

On this fixed-targeting interval, $\frac{\Phi(L)}{\phi} = \Psi_T(L)$, where

$$\Psi_T(L) := \frac{R_T^2 P_T(L)}{A_T(L)^2} + \sum_{\ell \in U} \frac{\pi_\ell}{a_\ell(L)}, \qquad P_T(L) := \sum_{\ell \in T} \pi_\ell a_\ell(L).$$

The second term is weakly decreasing in L, because $\frac{1}{a_\ell(L)} = \frac{1}{L} + k_\ell$. Since $R_T > 0$, it remains to show that $\frac{P_T(L)}{A_T(L)^2}$ is strictly decreasing.

For each ℓ, $a_\ell'(L) = \frac{1}{(1+Lk_\ell)^2}$, $\qquad \frac{a_\ell'(L)}{a_\ell(L)} = \frac{1}{L(1+Lk_\ell)} =: b_\ell(L)$. Since $k_1 > \cdots > k_d$, the sequence $b_\ell(L)$ is weakly increasing in ℓ. Also, $\pi_\ell = r_\ell m_\ell$, and Assumption 3 implies that r_ℓ is weakly decreasing in ℓ. For $\ell \in T$, define $\omega_\ell(L) := \frac{m_\ell a_\ell(L)}{A_T(L)}$. Then

$$\frac{A_T'(L)}{A_T(L)} = \sum_{\ell \in T} \omega_\ell(L) b_\ell(L), \qquad \frac{P_T'(L)}{P_T(L)} = \frac{\sum_{\ell \in T} \omega_\ell(L) r_\ell b_\ell(L)}{\sum_{\ell \in T} \omega_\ell(L) r_\ell}.$$

Because r_ℓ is weakly decreasing in ℓ and $b_\ell(L)$ is weakly increasing in ℓ, their weighted covariance under the weights $\omega_\ell(L)$ is nonpositive. Hence $\frac{P_T'(L)}{P_T(L)} \le \frac{A_T'(L)}{A_T(L)}$. Since $a_\ell'(L) > 0$ for every $\ell \in T$ and $T \neq \emptyset$, we have $A_T'(L) > 0$. Therefore

$$\frac{d}{dL} \log \left(\frac{P_T(L)}{A_T(L)^2} \right) = \frac{P_T'(L)}{P_T(L)} - 2\frac{A_T'(L)}{A_T(L)} \le -\frac{A_T'(L)}{A_T(L)} < 0.$$

Thus $\frac{P_T(L)}{A_T(L)^2}$ is strictly decreasing. Since $R_T > 0$, the first term in $\Psi_T(L)$ is strictly decreasing. The second term is weakly decreasing, so $\Psi_T(L)$, and therefore $\Phi(L)$, is strictly decreasing on every interval on which the targeted set is constant.

It remains to verify that cutoff changes do not affect the monotonicity argument. Take any $L_2 > L_1 > 0$, and use the finite partition above. On each subinterval with nonempty interior and constant targeted set, the preceding argument shows that Φ is strictly decreasing. At each common endpoint, Φ is continuous, since w_B, q_ℓ, and a_ℓ are continuous. Thus Φ cannot increase at cutoff changes. Chaining the inequalities across the finite partition gives $\Phi(L_2) < \Phi(L_1)$. Hence Φ is strictly decreasing on $(0, \infty)$.

Therefore, the spillover-consistency equation $1 = \Phi(L)$ has at most one solution. Given L, the budget equation uniquely determines $w_B(L)$, and therefore determines $z = Lw_B(L)$, the survival profile $\mathbf{q}$, and the induced effort profile. Hence the surviving-neighbour simultaneous game has at most one type-symmetric equilibrium. Existence follows from Proposition 7, so the equilibrium is unique.

Assumption 3 is used only in the covariance step, where it makes r_ℓ weakly decreasing while $b_\ell(L)$ is weakly increasing. Without this condition, the first term in $\Psi_T(L)$ need not be monotone. ■

Having characterized the simultaneous benchmark, we now compare it with the sequential problem. Under this specification, the sequential objective is

$$\widetilde{W}(\mathbf{q}) := \sum_{\ell=1}^{d} m_\ell q_\ell x_\ell(\mathbf{q}) = \alpha \sum_{\ell=1}^{d} m_\ell q_\ell^2 \big[1 + \widetilde{L}(\mathbf{q}) k_\ell\big].$$

Since $\mathcal{Q}(B)$ is compact and $\widetilde{W}$ is continuous, a sequential optimum exists.

Lemma 4 (Sequential marginal cost under surviving-neighbor spillovers) *For every feasible survival profile* $\mathbf{q}$*,*

$$\frac{1}{m_\ell}\frac{\partial \widetilde{W}(\mathbf{q})}{\partial q_\ell} = 2x_\ell(\mathbf{q}) + 2\widetilde{\eta}(\mathbf{q})x_\ell(\mathbf{q})r_\ell,$$

where $r_\ell := \frac{\pi_\ell}{m_\ell}$ *and*

$$\widetilde{\eta}(\mathbf{q}) := \frac{\phi\widetilde{C}(\mathbf{q})}{1-\phi\widetilde{\kappa}(\mathbf{q})}, \qquad \widetilde{C}(\mathbf{q}) := \sum_{j=1}^{d} m_j k_j q_j^2, \qquad \widetilde{\kappa}(\mathbf{q}) := \sum_{j=1}^{d} \pi_j k_j q_j^2.$$

Proof. Write $\widetilde{W}(\mathbf{q}) = \alpha\sum_{j=1}^{d} m_j q_j^2 + \alpha\widetilde{L}(\mathbf{q})\widetilde{C}(\mathbf{q})$, where $\widetilde{C}(\mathbf{q}) := \sum_{j=1}^{d} m_j k_j q_j^2$. Also define $\widetilde{\mu}(\mathbf{q}) := \sum_{j=1}^{d} \pi_j q_j^2$ and $\widetilde{\kappa}(\mathbf{q}) := \sum_{j=1}^{d} \pi_j k_j q_j^2$. Since $\widetilde{L}(\mathbf{q}) = \frac{\phi\widetilde{\mu}(\mathbf{q})}{1-\phi\widetilde{\kappa}(\mathbf{q})}$, differentiating gives

$$\frac{\partial \widetilde{L}(\mathbf{q})}{\partial q_\ell} = \frac{2\phi\pi_\ell q_\ell[1+\widetilde{L}(\mathbf{q})k_\ell]}{1-\phi\widetilde{\kappa}(\mathbf{q})}.$$

Therefore,

$$\frac{1}{m_\ell}\frac{\partial \widetilde{W}(\mathbf{q})}{\partial q_\ell} = 2\alpha q_\ell[1+\widetilde{L}(\mathbf{q})k_\ell] + 2\alpha\frac{\phi\widetilde{C}(\mathbf{q})}{1-\phi\widetilde{\kappa}(\mathbf{q})}q_\ell[1+\widetilde{L}(\mathbf{q})k_\ell]\frac{\pi_\ell}{m_\ell}.$$

Using $x_\ell(\mathbf{q}) = \alpha q_\ell[1+\widetilde{L}(\mathbf{q})k_\ell]$, $r_\ell = \frac{\pi_\ell}{m_\ell}$, and $\widetilde{\eta}(\mathbf{q}) = \frac{\phi\widetilde{C}(\mathbf{q})}{1-\phi\widetilde{\kappa}(\mathbf{q})}$, we obtain $\frac{1}{m_\ell}\frac{\partial\widetilde{W}(\mathbf{q})}{\partial q_\ell} = 2x_\ell(\mathbf{q}) + 2\widetilde{\eta}(\mathbf{q})x_\ell(\mathbf{q})r_\ell$. ■

At a simultaneous allocation, effort equalization also equalizes the direct components of the sequential marginal cost. The amplification component here is proportional to $x_\ell(\mathbf{q})r_\ell$. Since $x_\ell(\mathbf{q}^s)$ is equal across targeted classes, relative belief weight r_ℓ, rather than $[1+L(\mathbf{q}^s)k_\ell]r_\ell$, determines the local comparison. This gives the proof of Proposition 5.

Proposition 9 (Coincidence under population-weighted beliefs) *Suppose Assumptions 1 and 2 hold and* $\pi_\ell = \frac{m_\ell}{n}$ *for every class* ℓ*. Under the surviving-neighbor specification, the simultaneous equilibrium policy and the sequential optimum are unique and coincide for every* $B \in (0, n)$*.*

Proof. Let $D(\mathbf{q}) := 1-\phi\widetilde{\kappa}(\mathbf{q}) > 0$. Under population-weighted beliefs, $\widetilde{C}(\mathbf{q}) = n\widetilde{\kappa}(\mathbf{q})$, so $\widetilde{W}(\mathbf{q}) = \frac{\alpha\sum_{\ell=1}^{d} m_\ell q_\ell^2}{D(\mathbf{q})}$.

Fix a simultaneous equilibrium $(\mathbf{q}^s, \mathbf{x}^s)$, which exists by Proposition 7, and write $w := \widetilde{W}(\mathbf{q}^s)$. Then $\widetilde{L}(\mathbf{q}^s) = \frac{\phi w}{\alpha n}$, and hence $x_\ell^s = \left(\alpha + \frac{\phi w k_\ell}{n}\right) q_\ell^s$. The simultaneous planner's optimality implies $\sum_{\ell=1}^{d} m_\ell x_\ell^s(q_\ell - q_\ell^s) \geq 0$ for every $\mathbf{q} \in \mathcal{Q}(B)$.

Using the expression for $\widetilde{W}$ we have

$$\begin{aligned} D(\mathbf{q})[\widetilde{W}(\mathbf{q}) - w] &= \sum_{\ell=1}^{d} m_\ell \left(\alpha + \frac{\phi w k_\ell}{n}\right)\left(q_\ell^2 - (q_\ell^s)^2\right) \\ &= 2\sum_{\ell=1}^{d} m_\ell x_\ell^s(q_\ell - q_\ell^s) \\ &\quad + \sum_{\ell=1}^{d} m_\ell \left(\alpha + \frac{\phi w k_\ell}{n}\right)(q_\ell - q_\ell^s)^2. \end{aligned}$$

The first sum in the last expression is nonnegative by simultaneous optimality. The second is strictly positive whenever $\mathbf{q} \neq \mathbf{q}^s$, since $\alpha > 0$ and $w \geq 0$. Since $D(\mathbf{q}) > 0$, we obtain $\widetilde{W}(\mathbf{q}) > \widetilde{W}(\mathbf{q}^s)$ for every feasible $\mathbf{q} \neq \mathbf{q}^s$. Thus $\mathbf{q}^s$ is the unique sequential optimum. This argument applies to *every* simultaneous equilibrium, so the simultaneous policy is also unique. ■

Corollary 4 (Strict gains from commitment under surviving-neighbor spillovers) *Suppose Assumptions 1, 2, and 3 hold. Let $(\mathbf{q}^s, \mathbf{x}^s)$ be a type-symmetric simultaneous equilibrium. If there exist $i < j$ in the targeted block T^s such that $r_i > r_j$, then $\mathbf{q}^s$ is not sequentially optimal. In particular, under link-based neighbor beliefs, any simultaneous equilibrium that targets at least two classes yields strictly more surviving activity than the sequential optimum.*

Proof. The first claim follows from Proposition 5. Under link-based neighbor beliefs,

$$\pi_\ell = \frac{k_\ell m_\ell}{\sum_{j=1}^d k_j m_j}, \qquad r_\ell = \frac{\pi_\ell}{m_\ell} = \frac{k_\ell}{\sum_{j=1}^d k_j m_j}.$$

Thus $i < j$ implies $k_i > k_j$, and therefore $r_i > r_j$. If at least two classes are targeted, there exist $i < j$ in T^s. Hence Proposition 5 applies, so $\mathbf{q}^s$ is not sequentially optimal. ■

We next characterize the global sequential allocation. Full removal is ruled out in this specification. At $q_\ell = 0$, the marginal cost of increasing class ℓ's survival probability is zero because its effort and its surviving-neighbor contribution are both zero. The following result makes this distinction from the baseline precise.

Proposition 10 (Degree ordering of sequential targeting under surviving-neighbor spillovers) *Suppose Assumptions 1, 2, and 3 hold. Let $\mathbf{q}^{\text{seq}}(B)$ be any sequential optimum under the surviving-neighbour specification. Then*

$$0 < q_1^{\text{seq}}(B) \leq q_2^{\text{seq}}(B) \leq \cdots \leq q_d^{\text{seq}}(B).$$

Thus higher-degree classes face weakly higher removal probabilities, but no class is fully removed.

Moreover, there exists an index $0 \leq \tau^{\text{seq}} \leq d$ such that

$$0 < q_\ell^{\text{seq}}(B) < 1 \quad \text{for } \ell \leq \tau^{\text{seq}}, \qquad q_\ell^{\text{seq}}(B) = 1 \quad \text{for } \ell > \tau^{\text{seq}}.$$

Within the partially targeted block, survival probabilities are strictly increasing as degree falls.

Proof. Rewrite the budget constraint as $\sum_{\ell=1}^d m_\ell q_\ell = n - B$. Since $\widetilde{W}$ is continuously differentiable and the constraints are linear, the first-order necessary conditions imply that, at any sequential optimum $\mathbf{q}^{\text{seq}}(B)$, there exists a Lagrange multiplier λ^{seq}. The per-capita marginal cost of survival equals λ^{seq} for interior classes, is at least λ^{seq} when $q_\ell^{\text{seq}}(B) = 0$, and is at most λ^{seq} when $q_\ell^{\text{seq}}(B) = 1$.

Let $\widetilde{L}^{\text{seq}} := \widetilde{L}(\mathbf{q}^{\text{seq}}(B))$ and $\widetilde{\eta}^{\text{seq}} := \widetilde{\eta}(\mathbf{q}^{\text{seq}}(B))$. By Lemma 4, the per-capita marginal cost of survival in class ℓ is $2\alpha q_\ell^{\text{seq}}(B)\big[1 + \widetilde{L}^{\text{seq}} k_\ell\big]\big[1 + \widetilde{\eta}^{\text{seq}} r_\ell\big]$.

Since $B < n$, at least one class has positive survival and hence a strictly positive per-capita marginal cost of survival. The first-order conditions imply that this marginal cost is at most λ^{seq}, with equality if the class is interior. Therefore $\lambda^{\text{seq}} > 0$.

If $q_\ell^{\text{seq}}(B) = 0$ for some class ℓ, its per-capita marginal cost is zero. This contradicts the first-order condition at the lower bound, which requires this marginal cost to be at least $\lambda^{\text{seq}} > 0$. Hence $q_\ell^{\text{seq}}(B) > 0$ for every ℓ.

Define $z_\ell := \frac{\lambda^{\text{seq}}}{2\alpha\left[1+\widetilde{L}^{\text{seq}}k_\ell\right]\left[1+\widetilde{\eta}^{\text{seq}}r_\ell\right]}$. The first-order conditions imply $q_\ell^{\text{seq}}(B) = \min\{1, z_\ell\}$. Indeed, if $0 < q_\ell^{\text{seq}}(B) < 1$, the per-capita marginal cost equals λ^{seq}, so $q_\ell^{\text{seq}}(B) = z_\ell$. If $q_\ell^{\text{seq}}(B) = 1$, the marginal cost is at most λ^{seq}, so $z_\ell \geq 1$. The lower bound does not bind, as shown above.

Take $i < j$. Then $k_i > k_j$, and Assumption 3 gives $r_i \geq r_j$. Since $\widetilde{L}^{\text{seq}} > 0$ and $\widetilde{\eta}^{\text{seq}} \geq 0$,

$$\left[1 + \widetilde{L}^{\text{seq}}k_i\right]\left[1 + \widetilde{\eta}^{\text{seq}}r_i\right] > \left[1 + \widetilde{L}^{\text{seq}}k_j\right]\left[1 + \widetilde{\eta}^{\text{seq}}r_j\right].$$

Hence $z_i < z_j$. Since $z \mapsto \min\{1, z\}$ is weakly increasing, $q_i^{\text{seq}}(B) \leq q_j^{\text{seq}}(B)$. Thus higher-degree classes have weakly lower survival probabilities.

The block representation follows directly. Since $q_\ell^{\text{seq}}(B) > 0$ for every ℓ, no class is fully removed. Since survival probabilities are weakly increasing in the class index, the partially targeted classes form an initial block. Thus there exists a cutoff $\tau^{\text{seq}} \in \{0, \ldots, d\}$ such that $0 < q_\ell^{\text{seq}}(B) < 1$ for $\ell \leq \tau^{\text{seq}}$, and $q_\ell^{\text{seq}}(B) = 1$ for $\ell > \tau^{\text{seq}}$. For any $i < j$ in the partially targeted block, $q_i^{\text{seq}}(B) = z_i < z_j = q_j^{\text{seq}}(B)$. Hence survival probabilities are strictly increasing in the class index within this block. ■

B.2 Numerical computation of the sequential benchmark

This subsection verifies the sequential benchmark reported in Table 1. The sequential problem is

$$\min_{\mathbf{q}\in\mathcal{Q}(B)} W^{\text{seq}}(\mathbf{q}) = \min_{\mathbf{q}\in\mathcal{Q}(B)} \left\{ \alpha \sum_{\ell=1}^{d} m_\ell q_\ell^2 [1 + L(\mathbf{q})k_\ell] \right\},$$

where $\mathcal{Q}(B) = \left\{\mathbf{q} \in [0,1]^d : \sum_{\ell=1}^d m_\ell(1 - q_\ell) = B\right\}$. Let $D(\mathbf{q}) := 1 - \phi\kappa(\mathbf{q}) > 0$ and define $P_v(\mathbf{q}) := D(\mathbf{q})[W^{\text{seq}}(\mathbf{q}) - v]$. After eliminating q_4 using the budget, the Hessian of P_v in the remaining coordinates $\mathbf{u} = (q_1, q_2, q_3)$ is affine in $\mathbf{u}$ and independent of v. For each ϕ in the table, we check in exact rational arithmetic that the leading principal minors of $\nabla^2_{\mathbf{u}} P_v - 100 I_3$ are positive at all six vertices of the feasible set. The same matrix is therefore positive definite throughout the feasible set. At $v = W^{\text{seq}}_{\min}$, the function P_v is nonnegative and equals zero at every sequential optimum. Strict convexity then implies that the sequential optimum is unique.

To check the reported rounding, we solve the stationary equations on the face indicated by the entries equal to one in the reported sequential profile. We round the independent coordinates to twelve decimal places and use the budget to recover the remaining coordinate exactly. Let $\widehat{\mathbf{q}}$ be the resulting feasible profile, set $v = W^{\text{seq}}(\widehat{\mathbf{q}})$, and let $g \leq 0$ be the minimum of the tangent to P_v at $\widehat{\mathbf{u}}$ over the six vertices. Since $D(\mathbf{q}) \geq 1 - \phi\bar{k} > 0$, convexity gives

$$v + \frac{g}{1 - \phi\bar{k}} \leq W^{\text{seq}}_{\min} \leq v.$$

The value bounds differ by less than 2×10^{-10} in each row. The Hessian bound also gives $\|\mathbf{u}^* - \widehat{\mathbf{u}}\| \leq \sqrt{\frac{-2g}{100}}$, where $\mathbf{u}^*$ gives the first three coordinates of the sequential optimum. This bound and the budget constraint verify the rounding of the reported survival probabilities.

B.3 Robustness to convex activity costs

The linear social cost in the baseline analysis is not essential for the local distortion result. Suppose instead that the planner evaluates survival profiles by $G_c(\mathbf{q}) = \sum_{\ell=1}^{d} m_\ell c(y_\ell(\mathbf{q}))$, where $y_\ell(\mathbf{q}) := q_\ell x_\ell(\mathbf{q})$, $c' > 0$, and $c'' \geq 0$. In the simultaneous problem, the planner takes effort as fixed. Hence, for every partially targeted class, the first-order condition is $c'(y_\ell)x_\ell = \lambda_{\text{sim}}$. Thus the simultaneous planner equalizes marginal-cost-weighted effort, rather than effort itself.

The sequential planner internalizes the induced effort response. Its per-capita marginal cost of survival is

$$\frac{1}{m_\ell}\frac{\partial G_c(\mathbf{q})}{\partial q_\ell} = 2c'(y_\ell(\mathbf{q}))x_\ell(\mathbf{q}) + \alpha\eta_c(\mathbf{q})[1 + L(\mathbf{q})k_\ell]r_\ell,$$

where $\eta_c(\mathbf{q}) := \frac{\phi \sum_{j=1}^{d} m_j c'(y_j(\mathbf{q}))k_j q_j^2}{1-\phi\kappa(\mathbf{q})}$. The first term is the direct marginal effect of survival on class ℓ's own activity. The second term is the amplification effect operating through the spillover index.

Now take two partially targeted classes a and b at the simultaneous equilibrium, and consider the budget-neutral perturbation $\mathbf{q}(\varepsilon) = \mathbf{q}^{\text{sim}} - \frac{\varepsilon}{m_a}\mathbf{e}_a + \frac{\varepsilon}{m_b}\mathbf{e}_b$. The directional derivative of the sequential objective at $\mathbf{q}^{\text{sim}}$ is

$$\frac{d}{d\varepsilon}G_c(\mathbf{q}(\varepsilon))\bigg|_{\varepsilon=0} = 2\big[c'(y_b)x_b - c'(y_a)x_a\big] - \alpha\eta_c(\mathbf{q}^{\text{sim}})\Big([1 + L(\mathbf{q}^{\text{sim}})k_a]r_a - [1 + L(\mathbf{q}^{\text{sim}})k_b]r_b\Big).$$

The first term is zero by the simultaneous first-order condition. Hence the sign is determined by the amplification score $\mathcal{A}_\ell(\mathbf{q}^{\text{sim}}) := [1 + L(\mathbf{q}^{\text{sim}})k_\ell]r_\ell$. If $\mathcal{A}_a(\mathbf{q}^{\text{sim}}) > \mathcal{A}_b(\mathbf{q}^{\text{sim}})$, then shifting enforcement toward class a strictly lowers the sequential objective. Under monotone neighbor-belief weights, this score is larger for higher-degree classes. Therefore, whenever at least two classes are partially targeted, a small budget-neutral shift of enforcement toward a higher-degree targeted class lowers the planner's objective under commitment. So the distortion comes from differential network amplification, not from the linearity of the planner's objective.